Theoretical Seminar@NCTS
Mar 9, 2007

Carrier-Mediated Ferromagnetism

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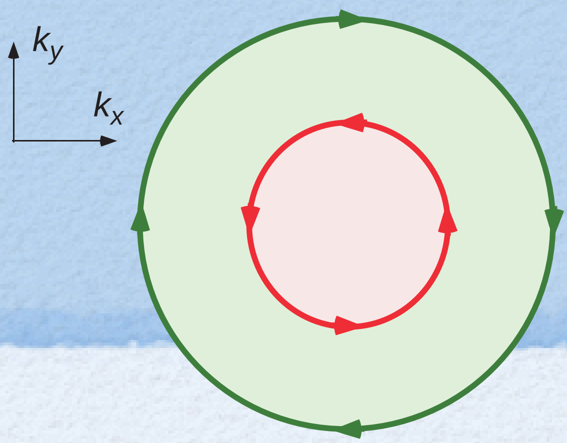
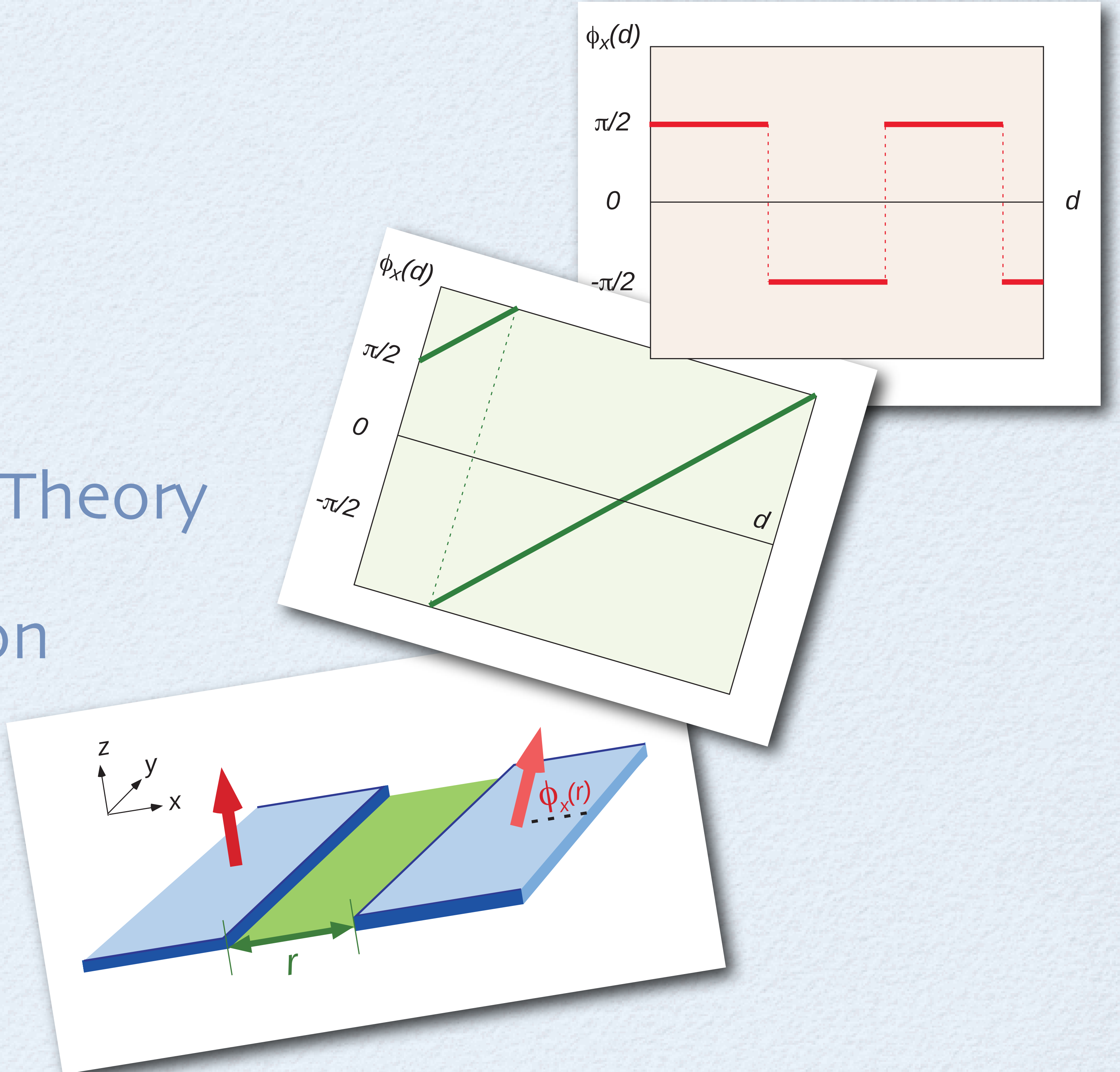


Outline

- Diluted Magnetic Semiconductors
- Two-component Ferromagnetism
- Building Spin-Wave Theory
- Spin-Wave Relaxation
- Sprial Exchange
- Summary



That's how it started...

M.-F. Yang, S.-J. Sun & M.-C. Chang

Phys. Rev. Lett. 86, 5636 (2001)

J. Konig, HHL & A. H. MacDonald

Phys. Rev. Lett. 86, 5637 (2001)

Spiral exchange interaction in diluted magnetic semiconductor junction

S.-J. Sun, S.-S. Chen & HHL

Appl. Phys. Lett. 84, 2862 (2004)

Spin-wave relaxation in diluted magnetic semiconductors within the self-consistent Green's function approach

J. E. Bunder, S.-J. Sun & HHL

Appl. Phys. Lett. 89,072101(2006)

Noncollinear exchange coupling in a trilayer magnetic junction and its connection to Fermi surface topology

W.-M. Huang, C.-H. Chang & HHL

Phys. Rev. B 73, 241307(R) (2006)

Spiral exchange coupling in trilayer magnetic junction mediated by diluted-magnetic-semiconductor thin film

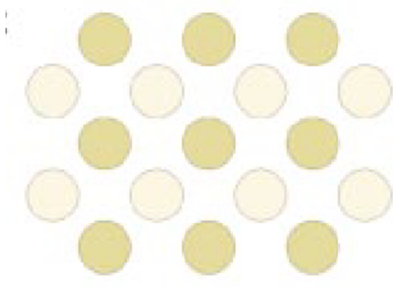
C.-H. Lin, HHL & T.-M. Hong

Appl. Phys. Lett. 89, 032503(2006)

Collaborators

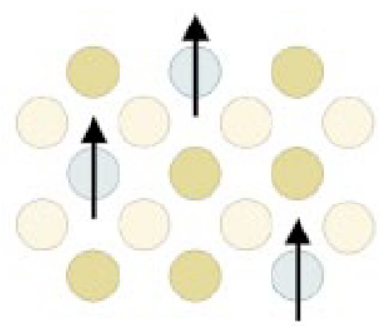
DILUTED MAGNETIC SEMICONDUCTORS

Ferromagnetic (Ga,Mn) As

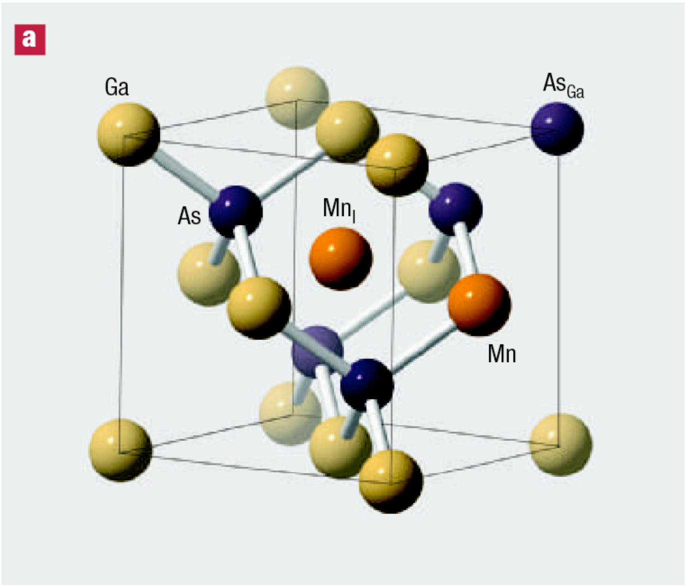


Ga
As

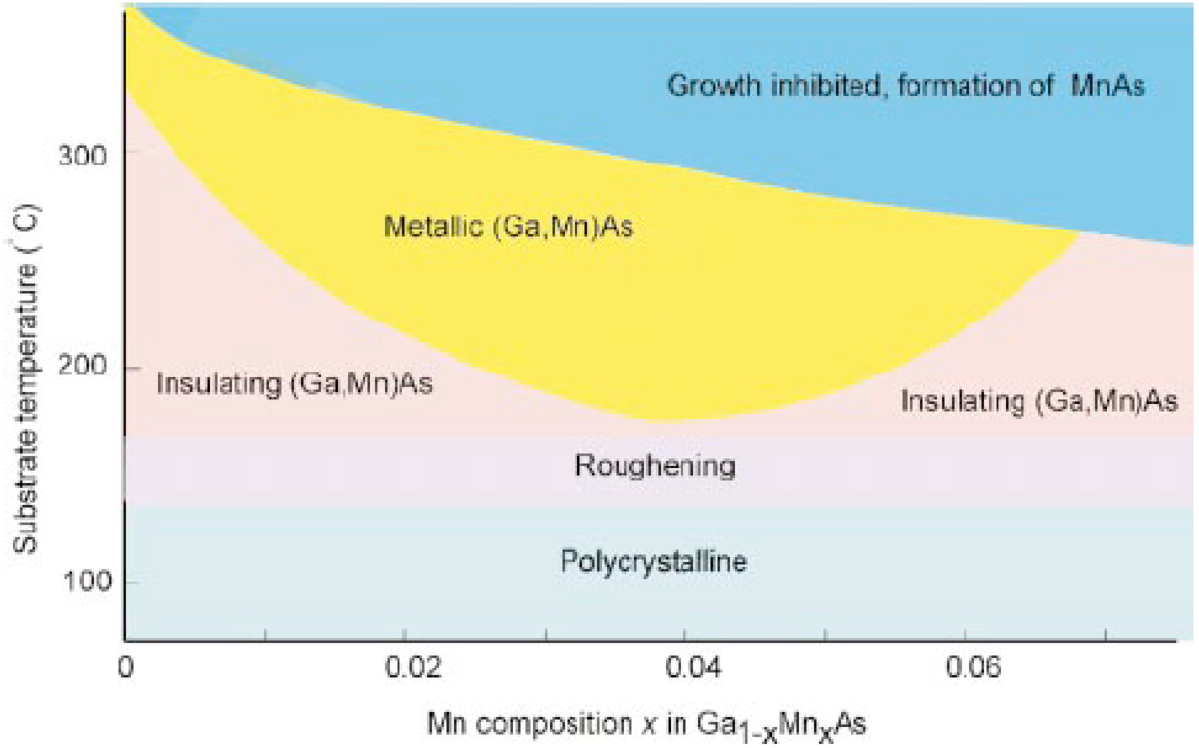
(Ga, Mn)
As



(Ga,Mn)As becomes ferromagnetic below Curie temperature T_c.

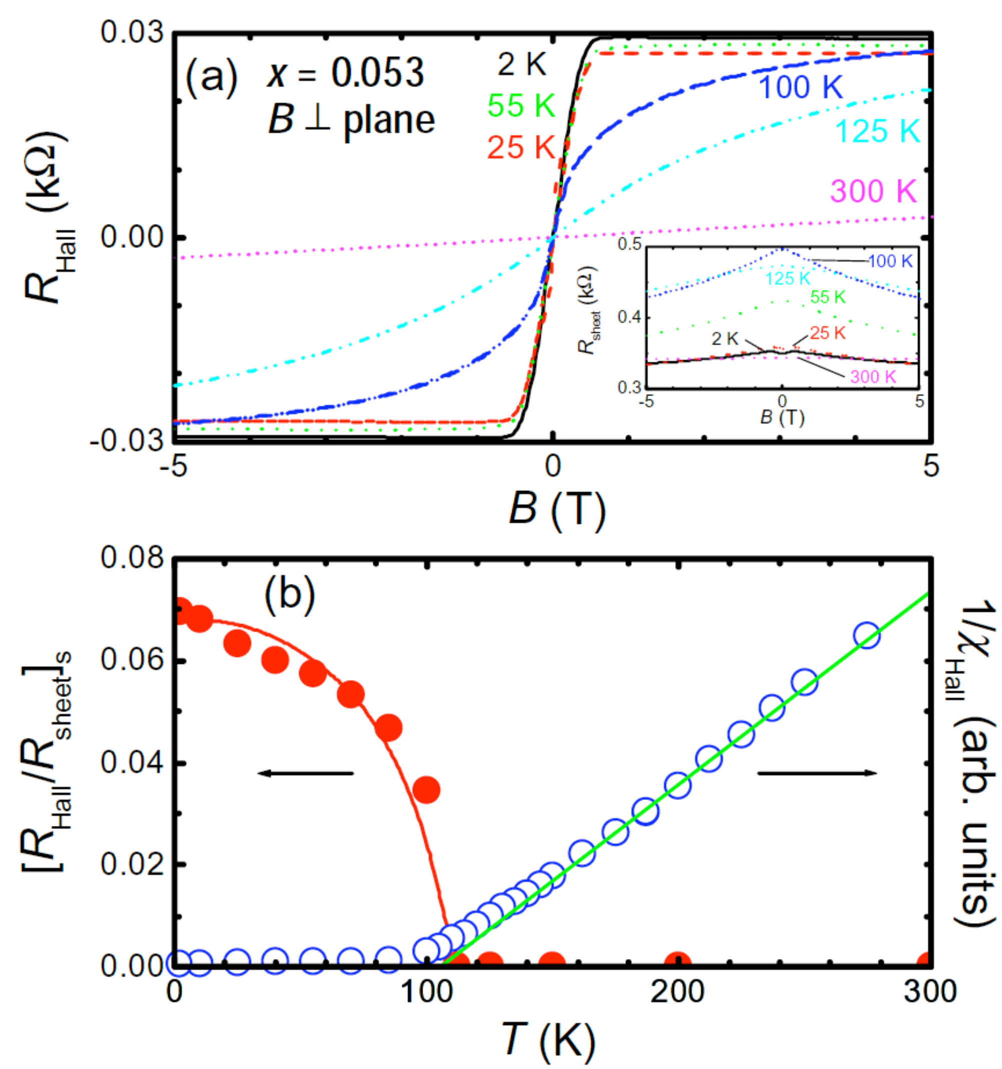


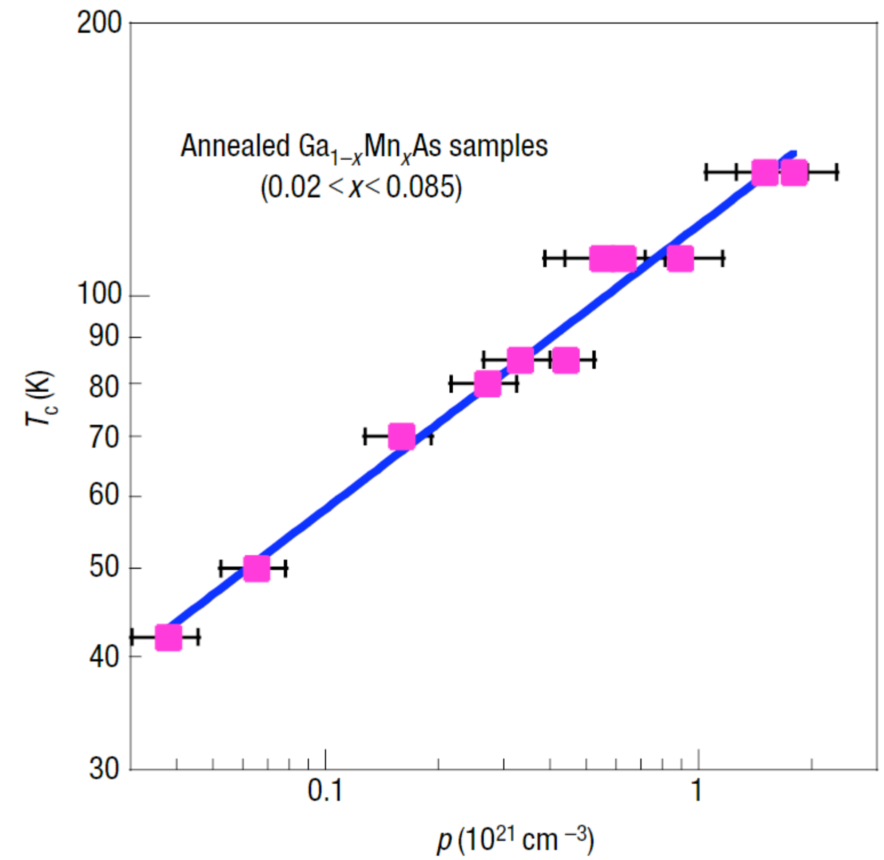
Zincblende structure of GaAs



Curie Temperature

Magnetization is measured through anomalous Hall effect.





The trend of Curie temperature can be fitted rather well by the empirical power law versus the carrier density,

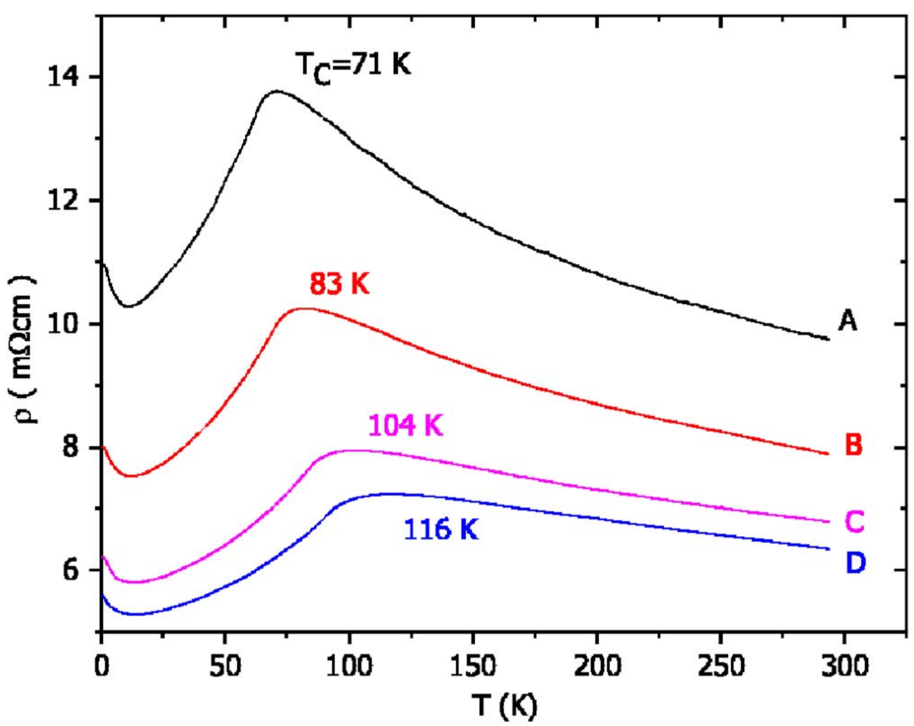
$$T_c \sim n_h^{1/3}$$

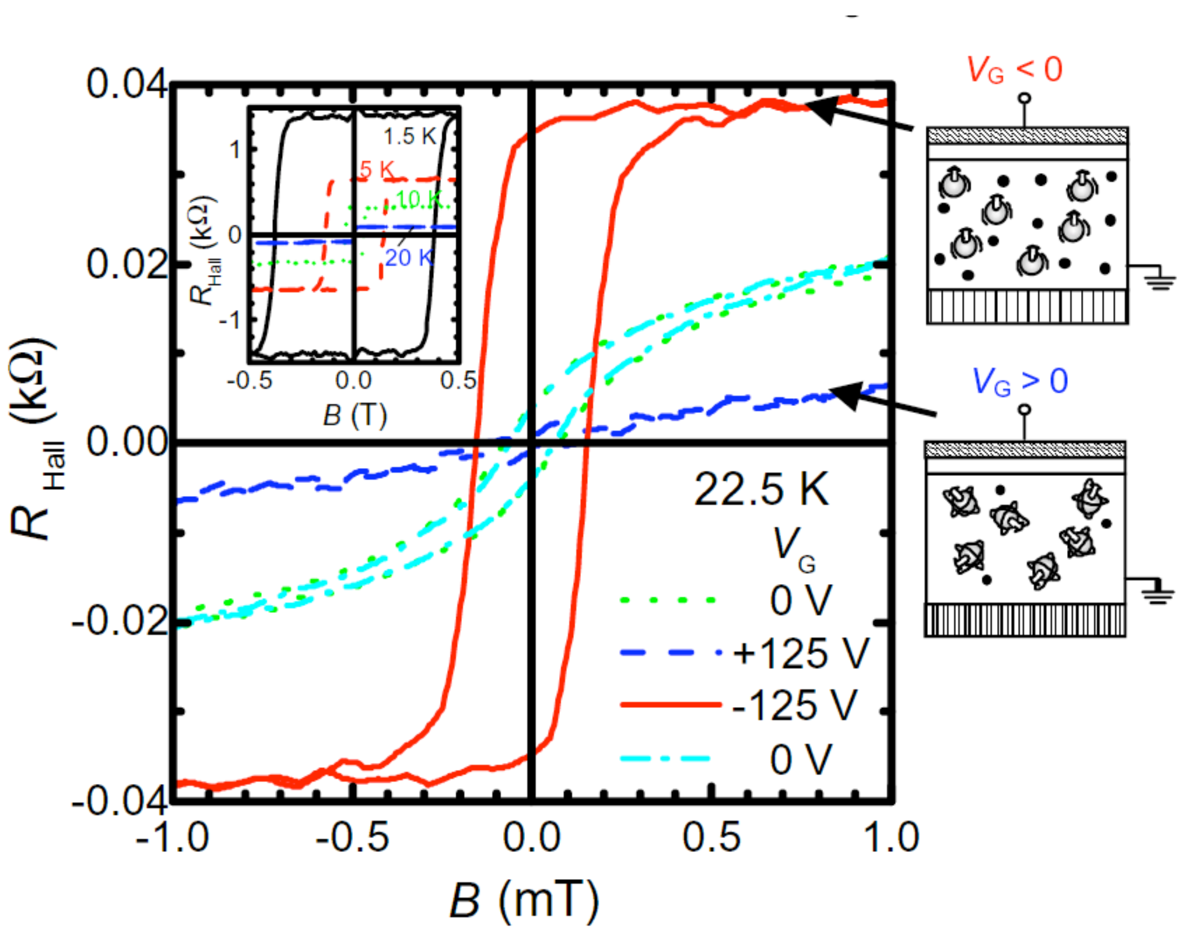
Transport and Field Effect

Ohno's Group
Nature 408, 944 (2000)

He, Yang, Ge, Wang, Dai, Wang
Appl. Phys. Lett. 87, 162506 (2005)

By varying the gate voltage, one can manipulate the concentration of itinerant carriers.

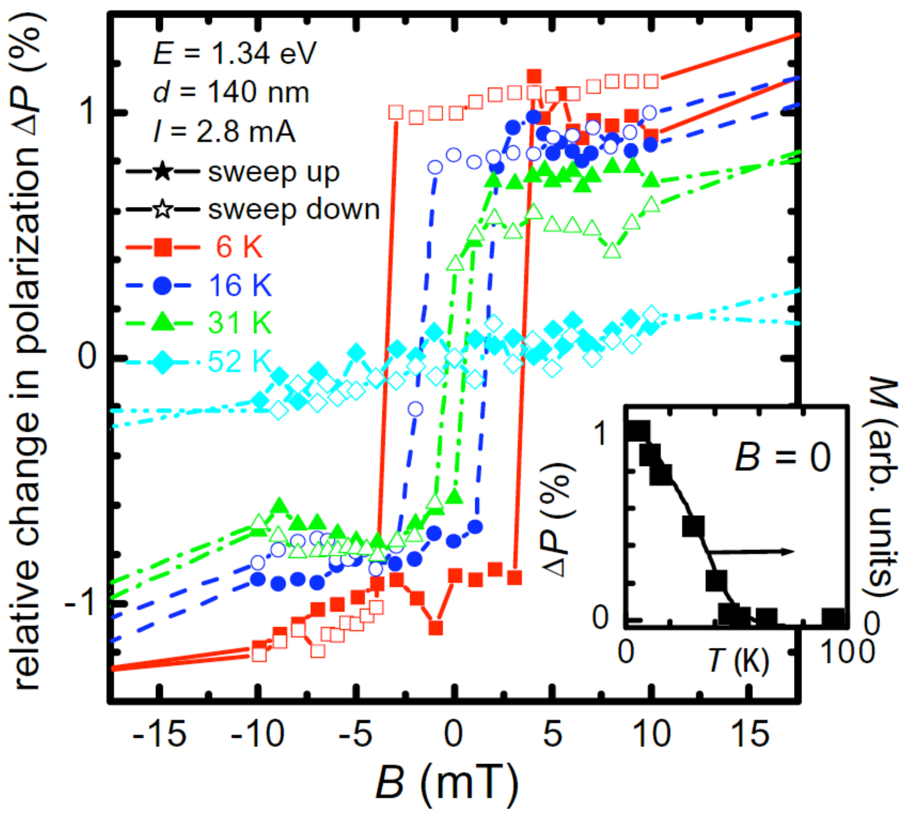




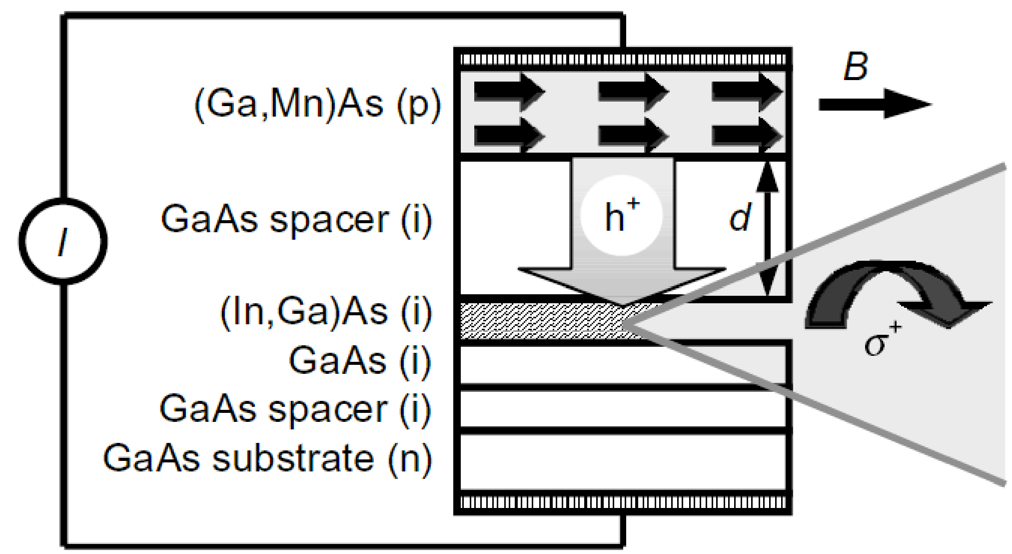
Resistivity shows pronounced peak around the Curie temperature. Is it the Fisher-Langer anomaly?

Spin Injection

Ohno's Group,
Nature 402, 790 (1999)



Polarized holes recombined with electrons, creating circularly polarized light by angular momentum transfer.



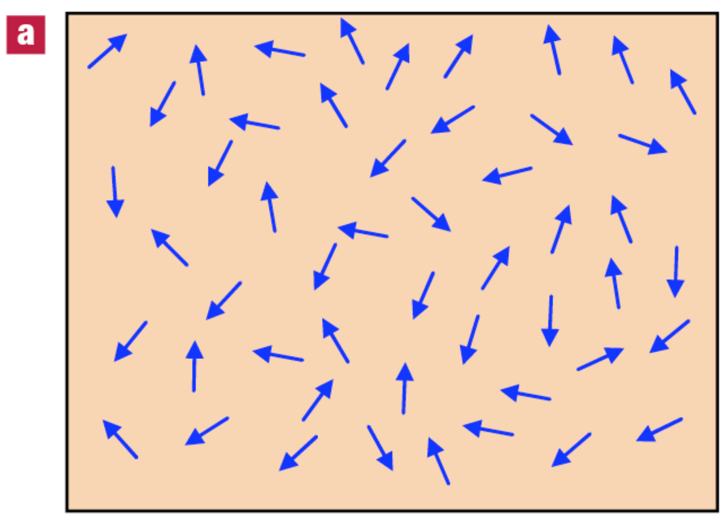
By measuring the intensity of the polarized light emission, we can estimate the efficiency of spin injection in the all-semiconductor setup.

TWO-COMPONENT FERROMAGNETISM

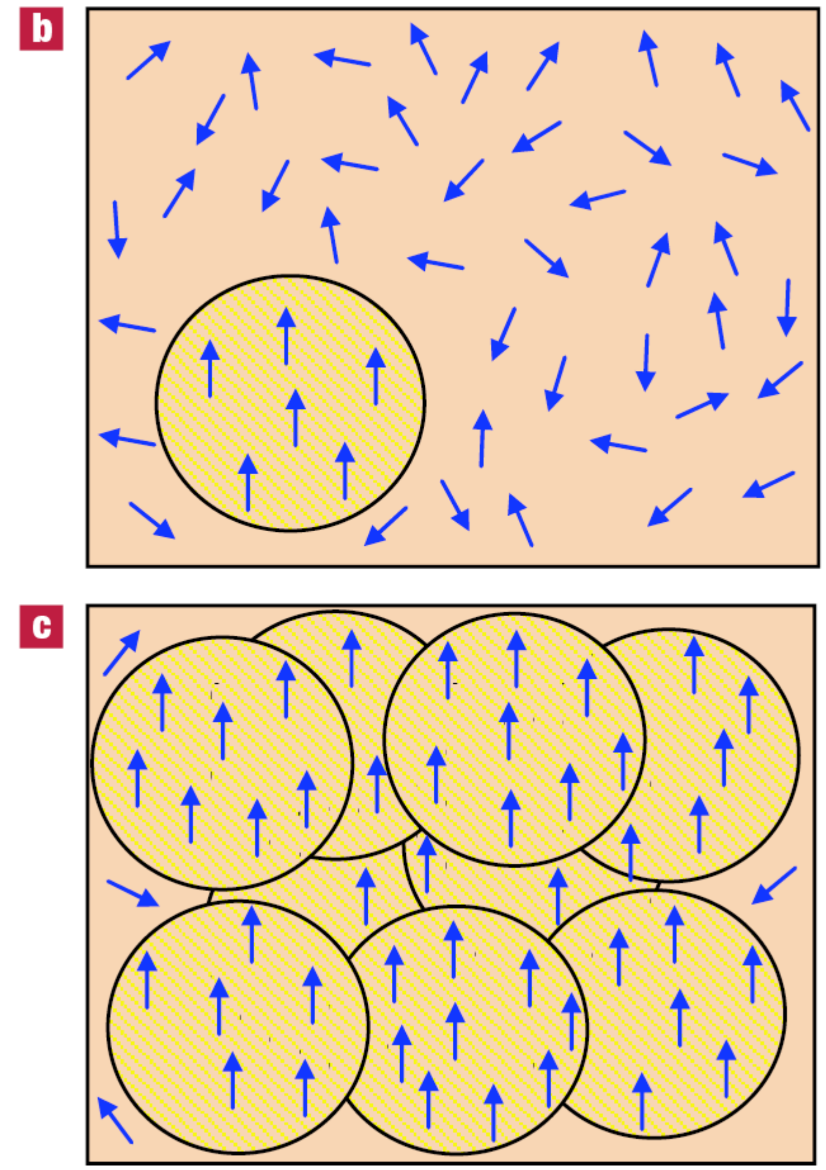
Carrier-Mediated Ferromagnetism

MacDonald et al.
Nature Materials 4, 195 (2005)

(a) At finite temperature, impurity spins prefer random orientations to maximize thermal entropy.



(b) The itinerant carriers like to align the impurity spins so that the kinetic energy is lowered.



(c) Delocalization of itinerant carriers leads to ferromagnetism.

Model

We start with the simplest model containing both itinerant and localized spins,

$$H = \int d^3r \left\{ \psi^\dagger(r) \left(-\frac{\nabla^2}{2m^*} - \mu \right) \psi(r) + J\,\boldsymbol{S}(r) \cdot \boldsymbol{\sigma}(r) \right\},$$

where J is the exchange coupling between impurity and itinerant spin densities, denoted by $\boldsymbol{S}(r), \boldsymbol{\sigma}(r)$ respectively,

$$\begin{aligned} \boldsymbol{S}(r) &= \sum_I \delta^3(r - R_I)\boldsymbol{S}_I, \\ \boldsymbol{\sigma}(r) &= \frac{1}{2}\psi^\dagger_\alpha(r)\boldsymbol{\tau}_{\alpha\beta}\psi_\beta(r). \end{aligned}$$

Of course, the realistic diluted magnetic semiconductors are beyond the simple Hamiltonian, which ignores several important pieces of physics: (1) realistic electronic band structure, (2) electron-electron interactions, (3) direct exchange between impurity spins.

Mean-Field Decomposition

Split the spin operators into mean-field and fluctuating parts,

$$\boldsymbol{S} = \langle S^z \rangle + \delta \boldsymbol{S}, \qquad \boldsymbol{\sigma} = \langle \sigma^z \rangle + \delta \boldsymbol{\sigma},$$

and dropping higher-order fluctuations $\delta\boldsymbol{S}\cdot\delta\boldsymbol{\sigma}$, the exchange coupling is approximated by

$$\begin{aligned} \boldsymbol{S}\cdot\boldsymbol{\sigma} &\approx \langle S^z \rangle \langle \sigma^z \rangle + \langle S^z \rangle \delta\boldsymbol{\sigma} + \langle \sigma^z \rangle \delta \boldsymbol{S} \\ &= -\langle S^z \rangle \langle \sigma^z \rangle + \langle S^z \rangle \sigma^z + \langle \sigma^z \rangle S^z, \end{aligned}$$

Dropping the constant in the first term, the mean-field Hamiltonian $H_{MF} = H_h + H_I$ is

$$\begin{aligned} H_h &= \int d^3r\, \psi_\sigma^\dagger (\epsilon_\sigma - \mu) \psi_\sigma, \\ H_I &= \int d^3r\, J \langle \sigma^z \rangle S^z, \end{aligned}$$

where the spectrums for the itinerant carriers are split by the exchange coupling, $\epsilon_\sigma = p^2/2m^* + (\sigma/2) J \langle S^z \rangle$.

Self-Consistency (I)

For notational convenience, introduce the polarization for both components of spins,

$$\alpha_I = \frac{1}{n_I S}\langle S^z\rangle, \qquad \alpha_h = -\frac{2}{n_h}\langle \sigma^z\rangle.$$

It is clear that the polarizations are always between zero and one, $0 \le \alpha_I, \alpha_h \le 1$. The minus sign is introduced in the second equation because the exchange coupling is antiferromagnetic.

With given polarization of the impurity spin α_I, one can proceed to compute the polarization of the itinerant carriers α_h. Note that the densities of itinerant carriers are

$$n_\nu = \int \frac{d^3k}{(2\pi)^3}\frac{1}{e^{\beta(\epsilon_\nu-\mu)}+1},$$

and the chemical potential μ is determined by keeping the total density of the itinerant carriers $n_\uparrow + n_\downarrow = n_h$ constant.

Self-Consistency (II)

Once the chemical potential is solved, the polarization of the itinerant carriers is

$$\alpha_h(\alpha_I, T) = \frac{n_\downarrow(\alpha_I, T) - n_\uparrow(\alpha_I, T)}{n_h}.$$

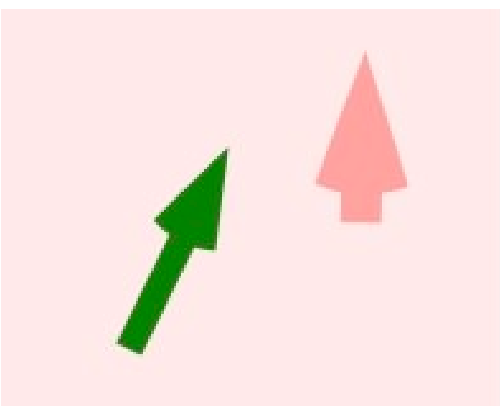
In general, $\alpha_h(\alpha_I, T)$ do not have a simple analytical form in terms of the impurity spin polarization α_I and the temperature T. To complete the self-consitency, we now compute the polarization of the impurity α_I with a given polarization α_h. The calculation leads to the Brillouin function,

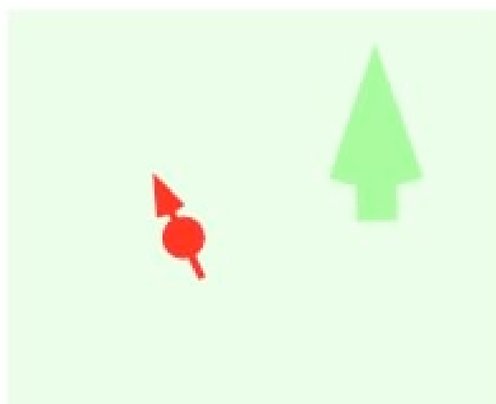
$$\alpha_I(\alpha_h, T) = B_S\left[\left(\frac{J n_h S}{2kT}\right)\alpha_h\right],$$

where the Brillouin function is $B_S(x) \equiv \frac{2S+1}{2S}\coth\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\coth\left(\frac{1}{2S}x\right)$. The above equations complete the self-consistency loop in Weiss mean-field theory.

Mean-Field Prediction

The polarization can be evaluated in mean-field limit by replacing all other spins with an effective magnetic field.





$$\langle S_z \rangle = \chi_C \, H = n_I \, \frac{S(S+1)}{3kT} \, g\mu_B H$$

$$\langle s_z \rangle = \chi_P \, h = \left[\frac{\chi_P}{(g^* \mu_B)^2} \right] \, g^* \mu_B h$$

The spin polarizations of Mn ions and itinerant holes under external magnetic field are described by **Curie** and **Pauli** susceptibilities.

Self-consistent equations at $T = T_c$

$$\langle S_z \rangle = n_I \, \frac{S(S+1)}{3kT_c} \, J \, \langle s_z \rangle$$

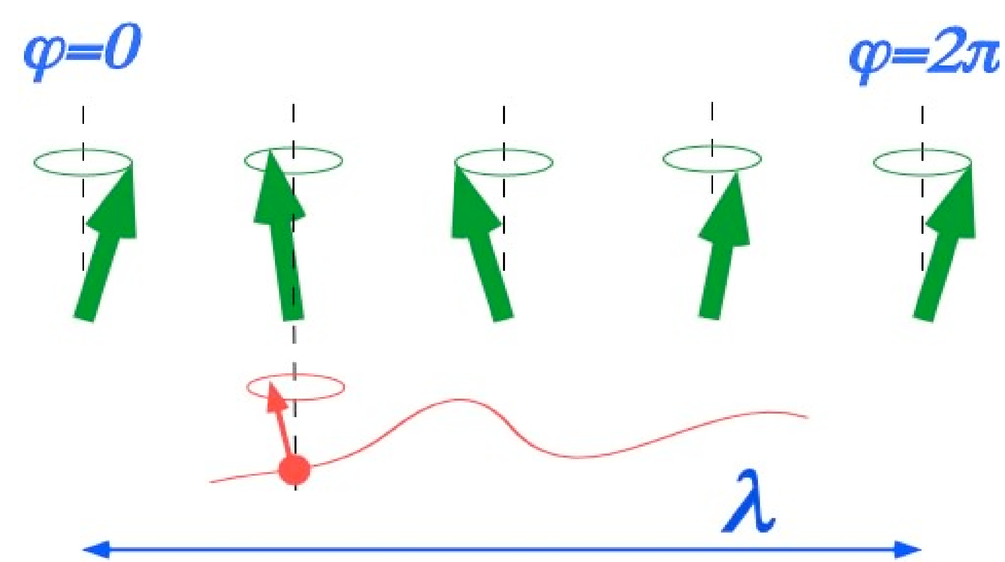
$$\langle s_z \rangle = \left[\frac{\chi_P}{(g^* \mu_B)^2} \right] \, J \, \langle S_z \rangle$$

$$kT_c = \frac{S(S+1)}{3} \, J^2 \, n_I \, \left[\frac{\chi_P}{(g^* \mu_B)^2} \right]$$

Building Spin-Wave Theory

Why Spin-Wave Theory?

Spin-wave theory -> the spatial fluctuations are inevitable once the SU(2) continuous symmetry is spontaneously broken.



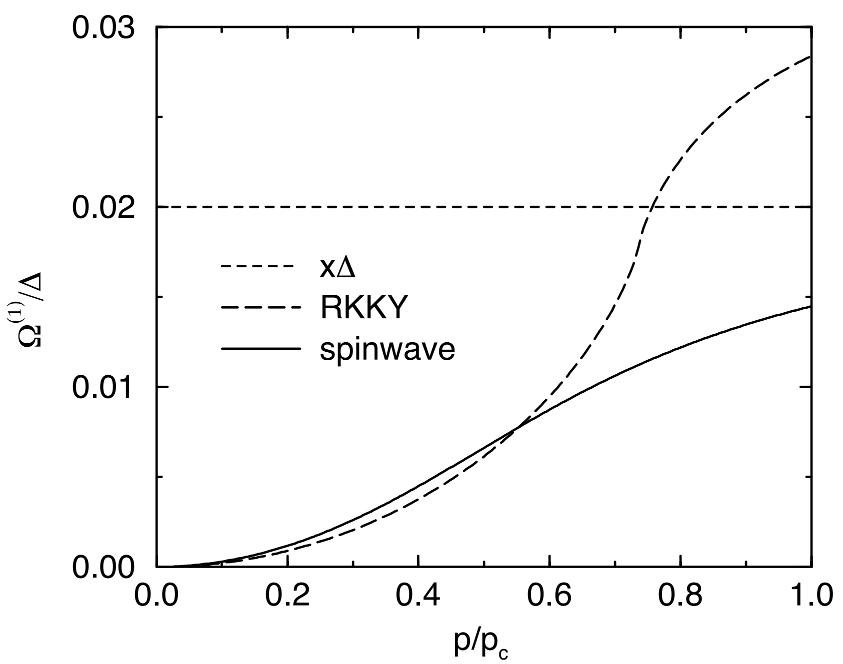


FIG. 1. Spin-wave dispersion for $c^* = 0.1\ \mathrm{nm}^{-3}$. The short wavelength limit is the mean-field result $x\Delta$. For comparison, we show also the result obtained from an RKKY picture.

$$
\begin{aligned}
D^{-1}(\vec{p},\Omega) \;&=\; -i\Omega + g\mu_B B + J\langle s_z\rangle \\
&\quad + \frac{n_I J^2 S}{2\beta V}\sum_{\vec{k},\nu} G_{\uparrow}^{MF}(\vec{k},\nu)\,G_{\downarrow}^{MF}(\vec{k}+\vec{p},\nu+\Omega)
\end{aligned}
$$

To include the spatial fluctuations, one needs to compute the spin-wave propagator D and the dispersion can be extracted from its pole.

Holstein-Primakov Boson

The spin-flip interactions $(S^+\sigma^- + S^-\sigma^+)$ were totally ignored in Weiss mean-field theory and the predicted *gapless* spin-wave excitations by Goldstone theorem are killed.

Making use of the path integral formalism, we can develop a spin-wave theory for the impurity spins by integrating out the itinerant ones. *However, the spin operator inside the path integral has the annoying* **Berry phase**. Therefore, some tweaking is in order...

$$
\begin{aligned}
S^+(r) &= \sqrt{2n_I S - b^\dagger(r)b(r)}\; b(r), \\
S^-(r) &= b^\dagger(r)\sqrt{2n_I S - b^\dagger(r)b(r)}, \\
S^z(x) &= n_I S - b^\dagger(r)b(r).
\end{aligned}
$$

In above, we introduce the Holsteiin-Primakov boson $b^\dagger(r), b(r)$ to represent the coarse-grained impurity spin density $\boldsymbol{S}(r)$.

Path Integral Formalism

Writing down the path integral for the HP bosons and the itinerant carriers,

$$
\begin{aligned}
Z &= \int D[\bar{z}z] \int D[\bar{\psi}\psi] e^{-S[\bar{\psi}\psi,\bar{z}z]}, \\
&= \int D[\bar{z}z] \int D[\bar{\psi}\psi] e^{-\int_0^\beta d\tau \int d^3r \ \mathcal{L}[\bar{\psi}\psi,\bar{z}z]}
\end{aligned}
$$

where $\mathcal{L} = \sum_\sigma [\bar{\psi}_\sigma(r,\tau)\partial_\tau\psi_\sigma(r,\tau) + \bar{z}(r,\tau)\partial_\tau z(r,\tau)] + H[\bar{\psi}\psi, \bar{z}z]$ is the Lagarangian density in the imaginary-time formalism.

Separating the action into two parts $S = S_z + S_\psi$,

$$
\begin{aligned}
S_z &= \int d\tau \int d^3r \ \bar{z}(r,\tau)\partial_\tau z(r,\tau), \\
S_\psi &= \int d\tau \int d^3r \int d\tau' \int d^3r' \ \bar{\psi}(r,\tau) G^{-1}(r,\tau;r',\tau')\psi(r',\tau'),
\end{aligned}
$$

with

$$
\boxed{G^{-1} = \left(\partial_\tau - \frac{\nabla^2}{2m^*} - \mu + \frac{J}{2}\boldsymbol{S}(\bar{z}z)\cdot\boldsymbol{\tau}\right)\mathbf{1},}
$$

where the short-hand notation is used, $\mathbf{1} = \delta(\tau-\tau')\delta^3(r-r')$.

Integrating Out Itinerant Carriers

Since the action is quadratic in $\overline{\psi}\psi$, we can integrate out the itinerant carriers,

$$\begin{aligned} Z &= \int D[\overline{z}z] e^{-S_z[\overline{z}z]} \int D[\overline{\psi}\psi] e^{-S_\psi[\overline{\psi}\psi\overline{z}z]} \\ &= \int D[\overline{z}z] e^{-S_z} \det G^{-1}(\overline{z}z) = \int D[\overline{z}z] e^{-S_{\text{eff}}[\overline{z}z]}, \end{aligned}$$

where $S_{\text{eff}}[\overline{z}z] = \left[\int d\tau \int d^3r \;\; \overline{z}(r,\tau)\partial_\tau z(r,\tau)\right] - \ln\det G^{-1}(\overline{z}z)$ is the effective action for the HP bosons.

Split G^{-1} into G_0^{-1}(z-independent) and δG^{-1}(z-dependent) parts,

$$\begin{aligned} G_0^{-1} &= \left(\partial_\tau - \frac{\nabla^2}{2m^*} - \mu + \frac{\Delta}{2}\tau^z\right)\mathbf{1}, \\ \delta G^{-1} &= \frac{J}{2}\sqrt{2n_I S}(z\tau^- + \overline{z}\tau^+) - \frac{J}{2}\overline{z}z\tau^z, \end{aligned}$$

where $\Delta = Jn_I S$ is the Zeeman splitting at zero temperature.

Expanding the Series...

Making use of the following identity to expand the determinant,

$$\begin{aligned}
\ln\det G^{-1} &= \mathrm{tr}\ln G^{-1} = \mathrm{tr}\ln(G_0^{-1} + \delta G^{-1}) \\
&= \mathrm{tr}\ln G_0^{-1} + \mathrm{tr}\ln(1 + G_0\delta G^{-1}) \\
&= \mathrm{tr}\ln G_0^{-1} - \mathrm{tr}\sum_{n=1}^{\infty}\left(-G_0\delta G^{-1}\right)^n .
\end{aligned}$$

Since δG^{-1} is at least linear in z, if we are interested in the quadratic terms of $\bar{z}, z$ (that is relevant to spin-wave propagation), we can truncate the series after the second term,

$$\begin{aligned}
S_{\mathrm{eff}}[\bar{z}z] &= \int d\tau \int d^3r \; \bar{z}(r,\tau)\partial_\tau z(r,\tau) - \ln\det G_0^{-1} \\
&\quad -\mathrm{tr}(G_0\delta G^{-1}) + \frac{1}{2}\mathrm{tr}(G_0\delta G^{-1}G_0\delta G^{-1})
\end{aligned}$$

leaving out the spin-wave interactions from higher-order terms.

Spin-Wave Propagator (I)

The first term gives the same result as in Weiss MFT,

$$
\begin{aligned}
-\mathrm{tr}(G_0 \delta G^{-1}) &= \frac{J}{2}\int d\tau \int d^3r \ \sum_\sigma \sigma G_0^\sigma(r,\tau;r,\tau^+)\bar{z}(r,\tau)z(r,\tau) \\
&= \frac{J}{2}(n_\downarrow - n_\uparrow)\int d\tau \int d^3r \ \bar{z}(r,\tau)z(r,\tau).
\end{aligned}
$$

To go from the first to the second line, we recall the definition that $G_0^\sigma(r,\tau;r,\tau^+) = \langle T\psi_\sigma(r,\tau)\psi_\sigma^\dagger(r,\tau^+)\rangle = -\langle\psi_\sigma^\dagger(r,\tau)\psi_\sigma(r,\tau)\rangle = -n_\sigma$. The second term is slightly more complicated,

$$
\begin{aligned}
\frac{1}{2}\mathrm{tr}(G_0\delta G^{-1}G_0\delta G^{-1}) &= \frac{n_I J^2 S}{2}\int d\tau \int d^3r \int d\tau' \int d^3r' \\
& \quad G_0^\uparrow(r,\tau;r'\tau')\bar{z}(r',\tau')G_0^\downarrow(r',\tau';r,\tau)z(r,\tau).
\end{aligned}
$$

Collecting both contributions, the spin-wave propagator is

$$
D^{-1}(r',\tau';r,\tau) = (\partial_\tau + \frac{J n_h \alpha_h}{2})\mathbf{1} + \frac{n_I J^2 S}{2} G_0^\uparrow(r,\tau;r'\tau')G_0^\downarrow(r',\tau';r,\tau).
$$

Spin-Wave Propagator (II)

Or, the propagator would look more familiar in momentum space,

$$D(p, \nu_n) = \frac{-1}{i\nu_n - \Sigma_W - \Sigma_{sw}(p, \nu_n)},$$

where the self energy corrections from integrating out the itinerant carriers are,

$$\begin{aligned} \Sigma_W &= \frac{1}{2} J\, n_h\, \alpha_h = J\langle \sigma^z \rangle, \\ \Sigma_{sw}(p, \nu_m) &= \frac{n_I J^2 S}{2\beta} \sum_m \int \frac{d^3k}{(2\pi)^3} G_0^{\uparrow}(k, \omega_n) G_0^{\downarrow}(k+p, \omega_m + \nu_n). \end{aligned}$$

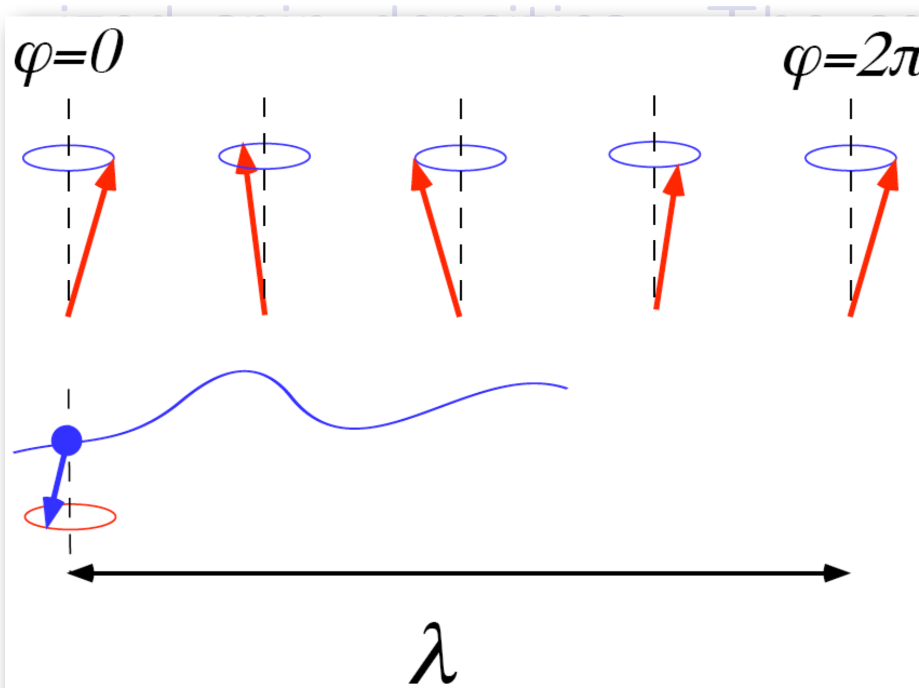
Note that, the constant part of the self energy Σ_W is the same as the Weiss mean-field theory at low temperatures, while $\Sigma_{sw}(p, \nu_n)$ carries frequency dependency, implying that there are *more than one collective excitations.*

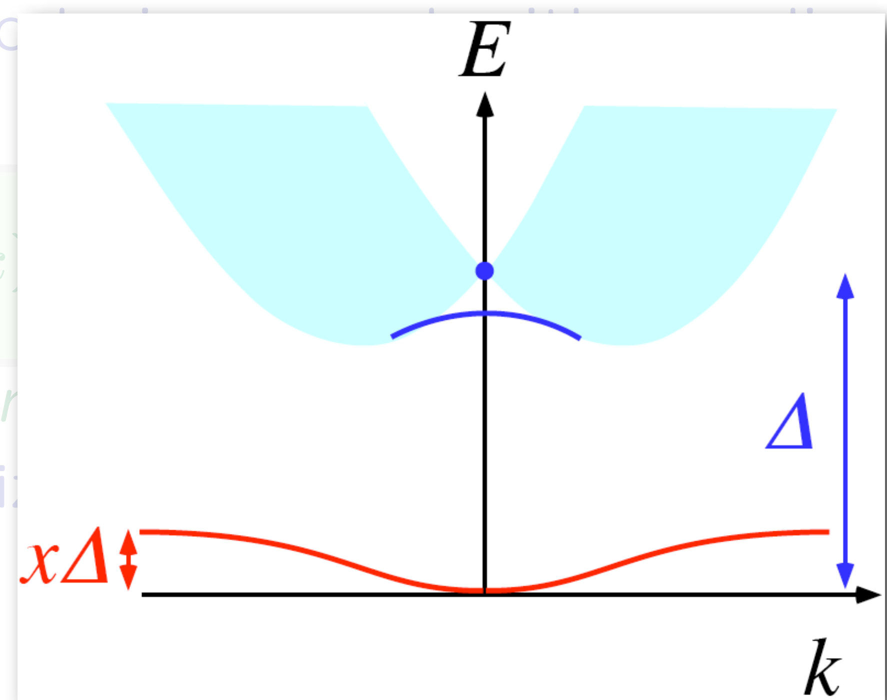
Collective Excitations

The dispersions of the collective excitations can be obtained by looking for the poles of the spin-wave propagator $D(p, \nu_n)$. After lengthy algebra, one will find two modes: the first mode is the usual spin wave of the localized spins,

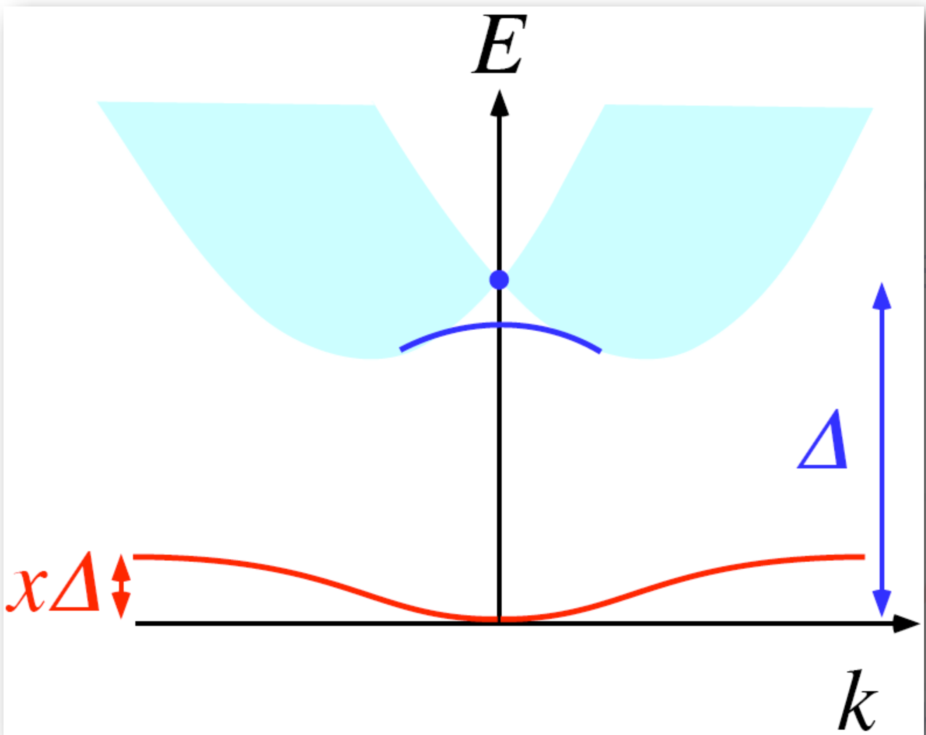
$$\boxed{E_1(k) = \frac{\gamma}{1-\gamma}\epsilon(k)\left[1 - \frac{4}{5}\frac{\epsilon_F}{\Delta}\right] + \mathcal{O}(k^4),}$$

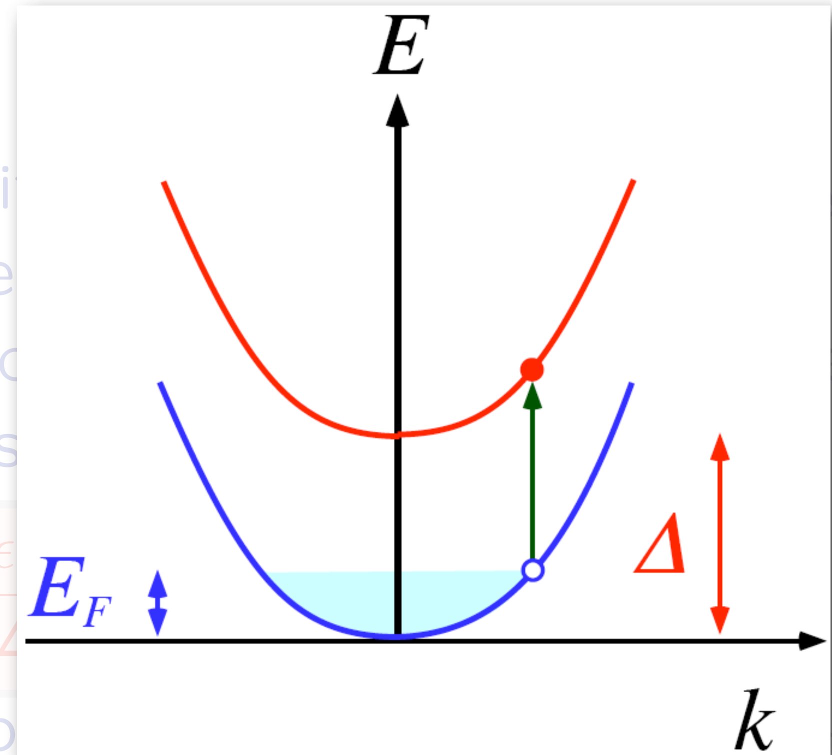
with $\gamma = n_h/2n_I S$ denotes the ratio between itinerant and local-





Collective Excitations





ized spin densities. The second mode is gapped with peculiar "banana"-shape dispersion,

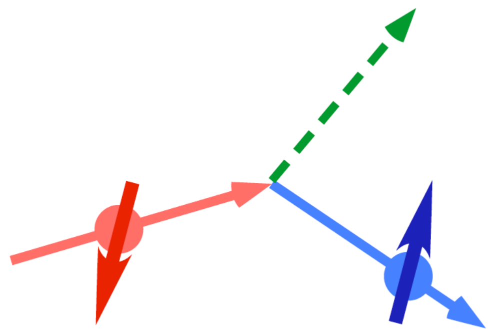
$$E_2(k) = \Delta(1-\gamma) - \frac{1}{\gamma(1-\gamma)}\epsilon(k)\left[\frac{4}{5}\frac{\epsilon_F}{\Delta} - \gamma\right] + \mathcal{O}(k^4).$$

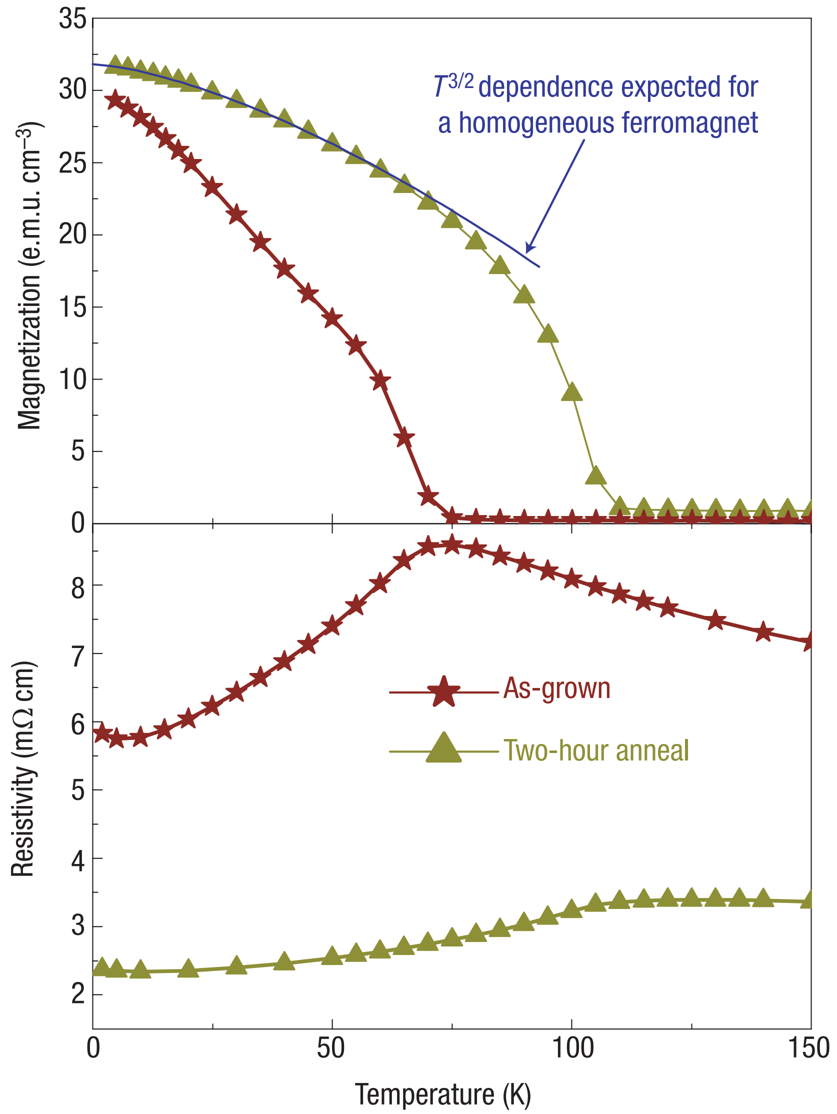
This mode comes from the *Stoner continuum of (electronic) magnons* that couple with the localized spins.

Spin Wave Relaxation

Transport Property

MacDonald et al.
Nature Materials 4, 195 (2005)





The conventional mean-field theory neglects the scattering between electronic magnons and spin waves. In order to address the transport issues, Green's function approach is necessary!!

What is the origin of the resistivity peak around the Curie temperature?

Green's Function Approach

(1) We start with the Zener model but keep ***both*** the itinerant and impurity spins.

(2) To describe the correlations between itinerant and impurity spins, we introduce the Green's functions:

$$\begin{aligned} D(r_1, r_2; t) &\equiv \langle\langle S^+(r_1, t); S^-(r_2, 0)\rangle\rangle \\ F(r_1, r_1', r_2; t) &\equiv \langle\langle \psi_\uparrow^\dagger(r_1, t)\psi_\downarrow(r_1', t); S^-(r_2, 0)\rangle\rangle. \end{aligned}$$

(3) Writing down the dynamical equations for the Green's function self-consistently.

(4) Solve the coupled differential equations and compute interested physical quantities, such as magnetization, spin-wave relaxation rate and so on.

Definition of Green's Function

The Green's function approach respects the spin kinematics and does not approximate spins as bosons. The thermal Green's function for the impurity spins is defined by

$$\begin{aligned} D(r_1, r_2; t) &\equiv \langle\langle S^+(r_1, t); S^-(r_2, 0)\rangle\rangle \\ &= -i\theta(t)\langle S^+(r_1, t)S^-(r_2, 0)\rangle + i\theta(-t)\langle S^-(r_2, 0)S^+(r_1, t)\rangle. \end{aligned}$$

As will become clear later, to complete the self-consistency, it is necessary to introduce another Green's function which describes the correlation between impurity and itinerant spins,

$$F(r_1, r_1', r_2; t) \equiv \langle\langle \psi_\uparrow^\dagger(r_1, t)\psi_\downarrow(r_1', t); S^-(r_2, 0)\rangle\rangle.$$

Time Evolution

Taking time derivative of the Green's function $D(r_1, r_2; t)$,

$$
\begin{aligned}
i\frac{\partial D}{\partial t} &= \delta(t)\langle S^+(r_1,0)S^-(r_2,0)\rangle + \Theta(t)\langle \frac{dS^+(r_1,t)}{dt} S^-(r_2,0)\rangle \\
&\quad -\delta(t)\langle S^-(r_2,0)S^-(r_1,0)\rangle + \Theta(t)\langle S^-(r_2,0)\frac{dS^+(r_1,t)}{dt}\rangle \\
&= \delta(t)\left\langle\left[S^+(r_1,0), S^-(r_2,0)\right]\right\rangle + i\left\langle\left\langle\frac{dS^+(r_1,t)}{dt}; S^-(r_2,0)\right\rangle\right\rangle.
\end{aligned}
$$

The first term consists of the equal-time commutator and can be easily calculated,

$$\delta(t)\langle[S^+(r_1,0), S^-(r_2,0)]\rangle = \delta(t)\delta^3(r_1 - r_2)\langle S^z(r_1)\rangle.$$

As for the second term, we need to derive the Heisenberg equation of the spin operator $S^+(r_1, t)$.

Heisenberg Equations

The time evolution of the spin operator $S^+(r_1,t)$ is,

$$i\frac{dS^+(r_1,t)}{dt} = [S^+(r_1,t), H] = [S^+(r_1,t), H_{ex}],$$

since the kinetic energy of the itinerant carriers is independent of the impurity spin and thus does not contribute.

The exchange interaction can be decomposed into three terms, $(J/2)S^+\sigma^- + (J/2)S^-\sigma^+ + JS^z\sigma^z$. The first term gives trivial zero and the dynamics is described by the remaining two terms,

$$\begin{aligned} i\frac{dS^+(r_1,t)}{dt} &= [S^+(r_1,t), H_{ex}] \\ &= \int d^3r \; JS^z(r_1,t)\sigma^+(r_1,t) - JS^+(r_1,t)\sigma^z(r_1,t). \end{aligned}$$

Now we are ready to put the operator back into the correlator.

Mean-Field Decomposition

Finally, we get the last piece of the puzzle,

$$i\left\langle\left\langle\frac{dS^{+}(r_1,t)}{dt};S^{-}(r_2,0)\right\rangle\right\rangle$$

$$= J\left\langle\left\langle S^{z}(r_1,t)\sigma^{+}(r_1,t);S^{-}(r_2,0)\right\rangle\right\rangle - J\left\langle\left\langle \sigma^{z}(r_1,t)S^{+}(r_1,t);S^{-}(r_2,0)\right\rangle\right\rangle,$$

$$\approx J\left\langle S^{z}(r_1)\right\rangle\left\langle\left\langle \sigma^{+}(r_1,t);S^{-}(r_2,0)\right\rangle\right\rangle - J\left\langle \sigma^{z}(r_1)\right\rangle\left\langle\left\langle S^{+}(r_1,t);S^{-}(r_2,0)\right\rangle\right\rangle,$$

$$= J\left\langle S^{z}(r_1)\right\rangle F(r_1,r_1,r_2;t) - J\left\langle \sigma^{z}(r_1)\right\rangle D(r_1,r_2;t).$$

Since the mean-field approximation is carried out at the level of equations of motion (not at the Hamiltonian level, as in Weiss mean-field theory), *it captures the correct spin kinematics and also includes the spatial fluctuations!!*

Self-Consistent EOM's

After some algebra, the self-consistent differential equations for the Green's functions are:

$$
\begin{aligned}
i\partial_t D(r_1, r_2; t) &= 2\langle S^z(r_1)\rangle \delta(t)\delta^3(r_1 - r_2) \\
&- J\langle \sigma^z(r_1)\rangle D(r_1, r_2; t) \\
&+ J\langle S^z(r_1)\rangle F(r_1, r_1, r_2; t),
\end{aligned}
$$

$$
\begin{aligned}
i\partial_t F(r_1, r_1', r_2; t) &= \left(\frac{\nabla^2_{r_1}}{2m^*} - \frac{J}{2}\langle S^z(r_1)\rangle \right) F(r_1, r_1', r_2; t) \\
&- \left(\frac{\nabla^2_{r_1'}}{2m^*} + \frac{J}{2}\langle S^z(r_1')\rangle \right) F(r_1, r_1', r_2; t) \\
&- \frac{J}{2}\langle \psi^\dagger_\downarrow(r_1)\psi_\downarrow(r_1')\rangle D(r_1, r_2; t) \\
&+ \frac{J}{2}\langle \psi^\dagger_\uparrow(r_1)\psi_\uparrow(r_1')\rangle D(r_1', r_2; t),
\end{aligned}
$$

Callen Formula

Yang, Chang, Sun
Phys. Rev. Lett. 86, 5636 (2001)

Konig, Lin, MacDonald
Phys. Rev. Lett. 86, 5637 (2001)

Yang, Chang and Sun pointed out that the spin-wave theory can be extended to finite temperature and the magnetization can be computed by the well-known ***Callen formula***,

$$\langle S^z \rangle = cS - c\Phi + \frac{(2S+1)c}{\left[(1+\Phi)/\Phi\right]^{2S+1} - 1},$$

$$\boxed{\Phi = \frac{1}{cV}\sum_p n(\Omega_p)}$$

We further notice that the connection to the conventional Weiss MFT is rather simple,

$$\langle S^z \rangle = c\{S - n(\Omega) + (2S+1)n[(2S+1)\Omega]\},$$

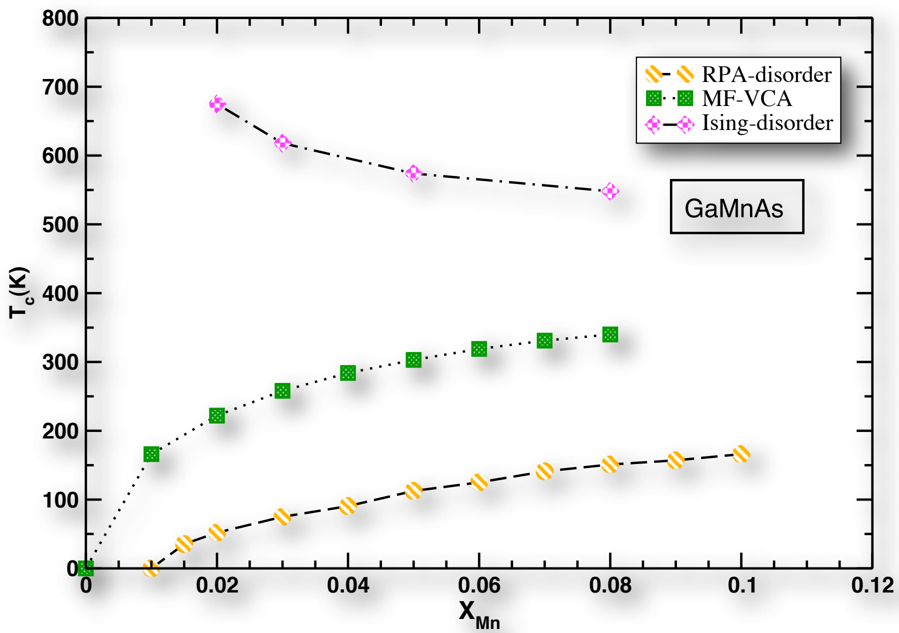
$$\boxed{n(\Omega) \equiv \frac{1}{cV}\sum_p n(\Omega_p)}$$

$$\boxed{k_B T_c = \frac{S(S+1)/3}{\lim_{\langle S^z \rangle, n^* \to 0}(1/V)\sum_{|\vec{p}|<p_c} \langle S^z \rangle / \Omega_p c^2}.}$$

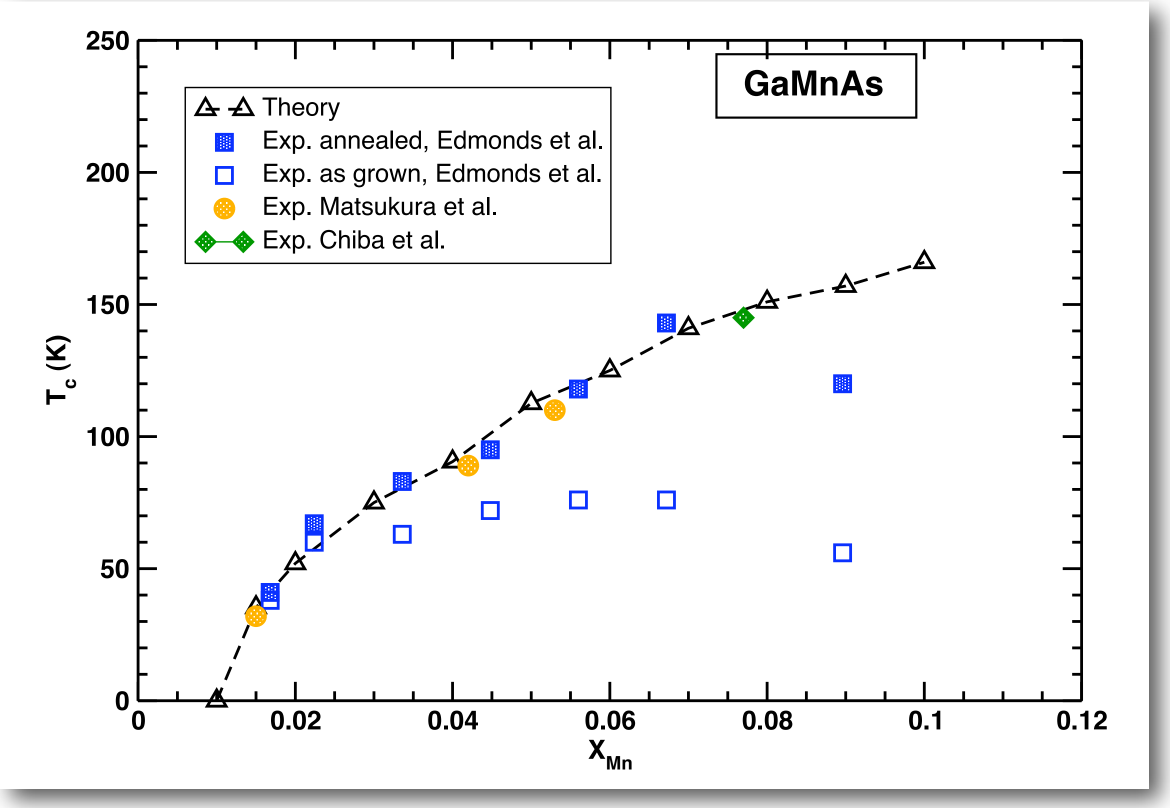
Estimate Curie Temperature?

To estimate the Curie temperature, it is crucially important to include the thermal fluctuations correctly. That is to say, one needs to respect the spin-wave kinematics:

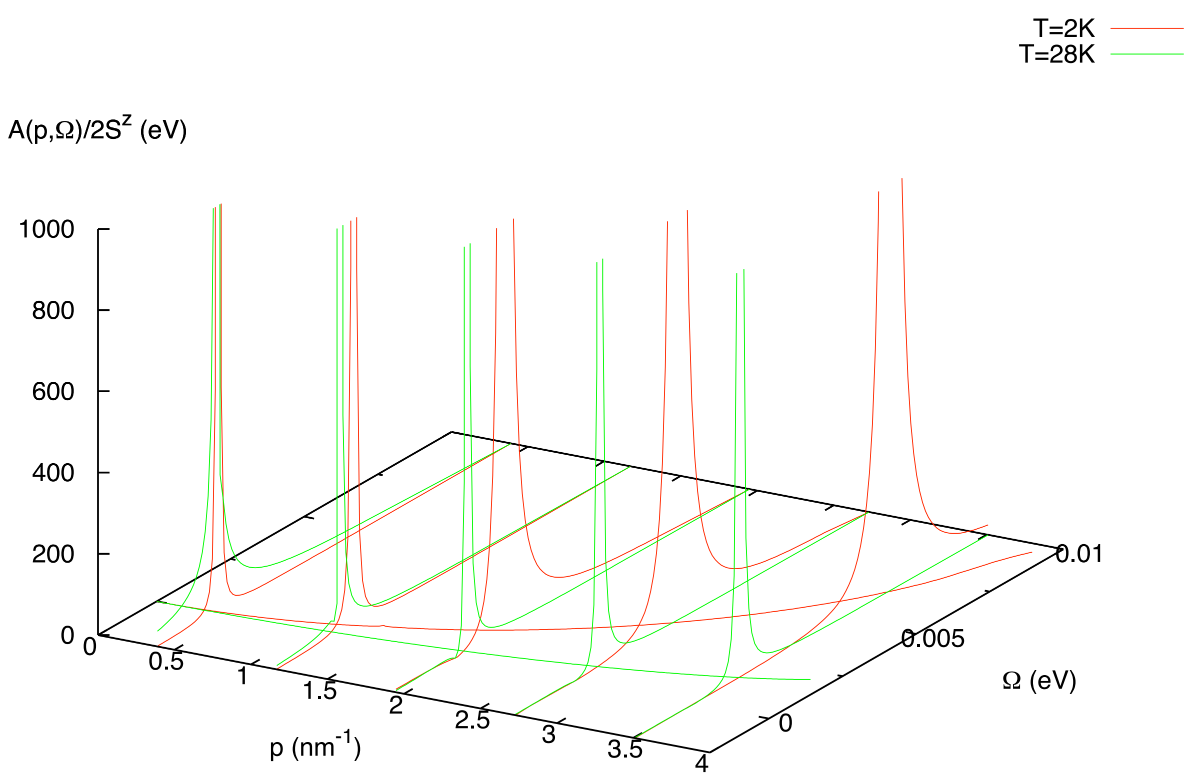
$$EG_{ij}(E) = 2\lambda_i \delta_{ij} + \left(\sum_l J_{lj} \lambda_l \right) G_{ij}(E) - \epsilon \left[\lambda_i \sum_l J_{il} G_{lj}(E) \right]$$



Bouzerar, Ziman, Kudrnovsky
Europhys. Lett. 69, 812 (2005)



Spin-Wave Relaxation



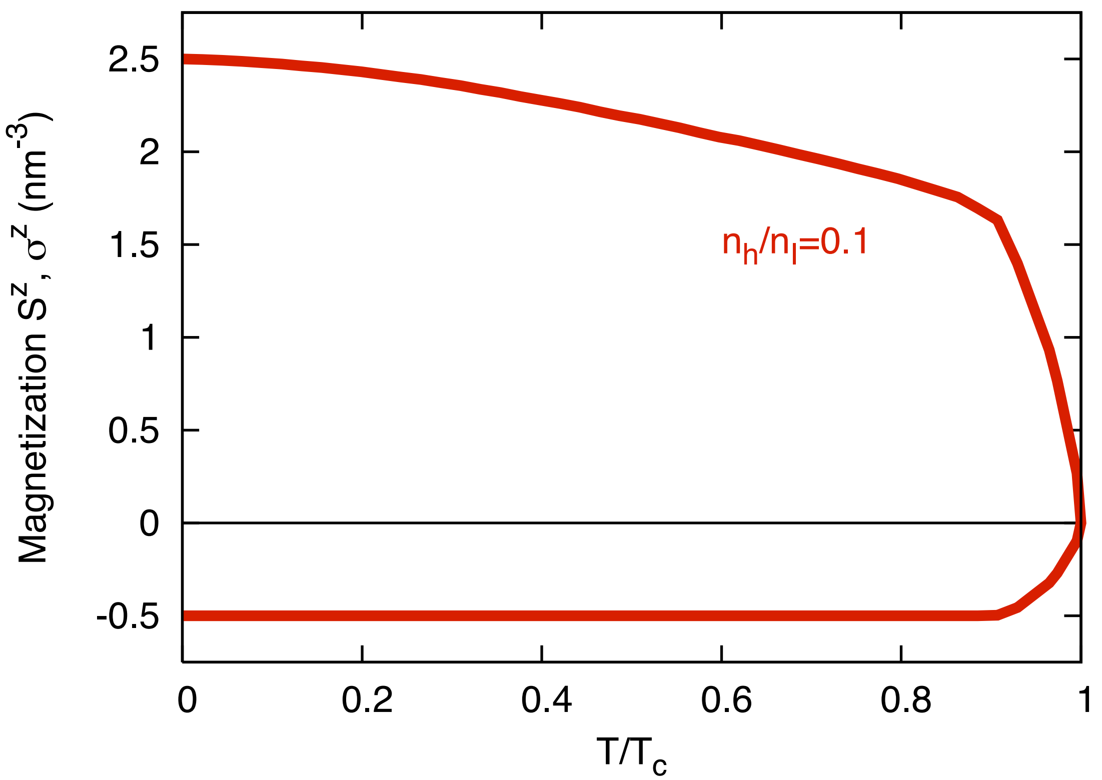
We can solve the spin-wave propagator first and compute its imaginary part to obtain the spin spectral function:

$$
\begin{aligned}
A(p,\omega) &= -\left(\frac{1}{\pi}\right)\mathrm{Im}D(p,\Omega), \\
&= 2\langle S^z\rangle\left(\frac{Z}{\pi}\right)\frac{\Gamma(p)}{(\Omega-\omega_p)^2+\Gamma(p)^2}
\end{aligned}
$$

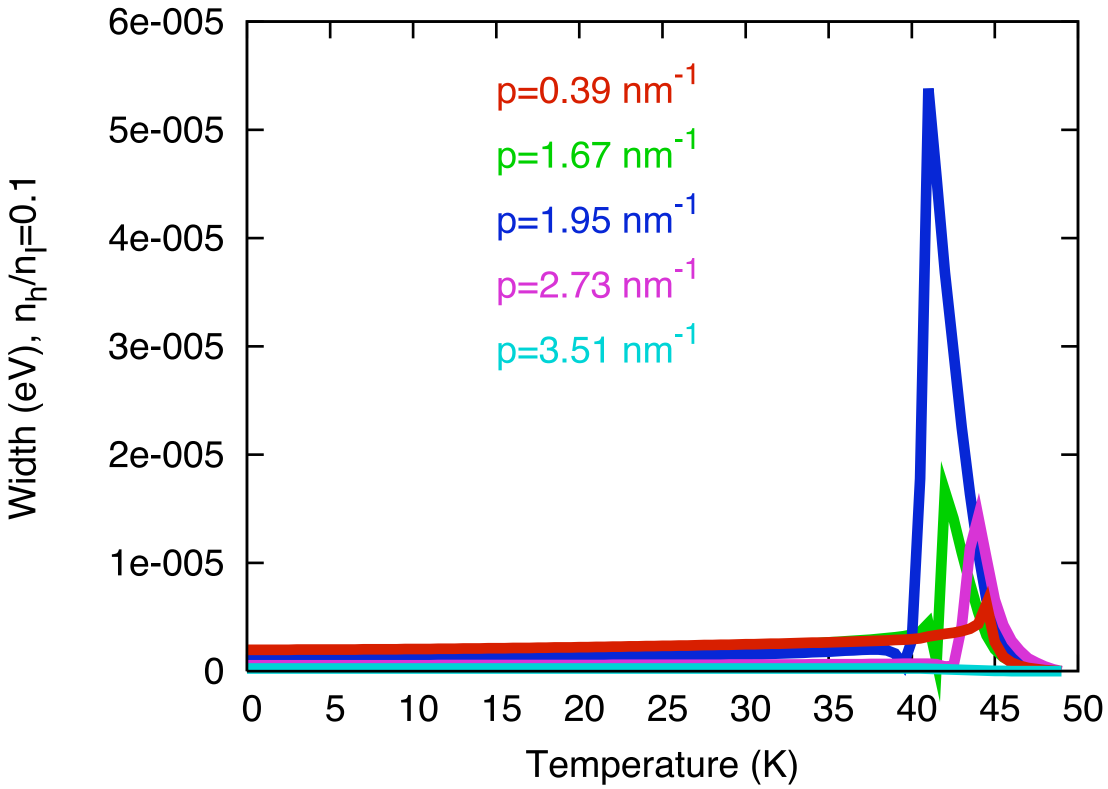
(1) The spin spectral function takes the Lorentzian shape with temperature-dependent halfwidth.

(2) The spectral function actually ***sharpens up*** when T increases!!

Anomalous T dependence (I)

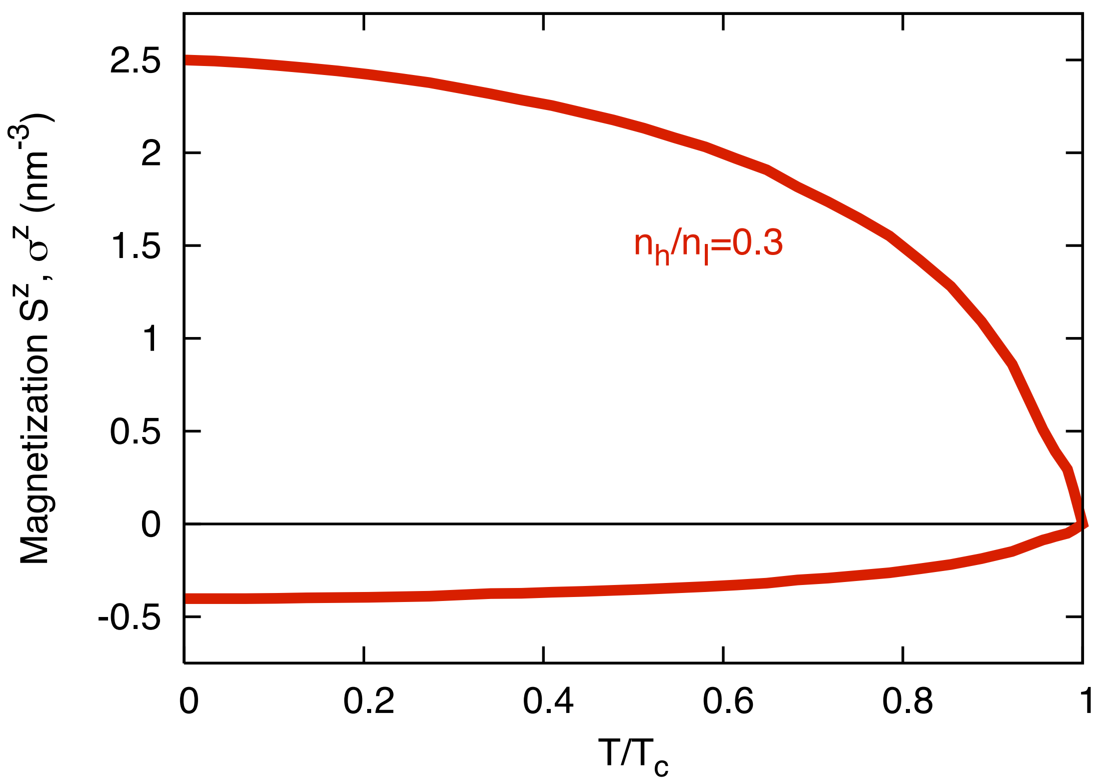


For the density ratio n_h/n_l=0.1, the magnetization curve shows rather unusual shape compared with the conventional Brillouin function.

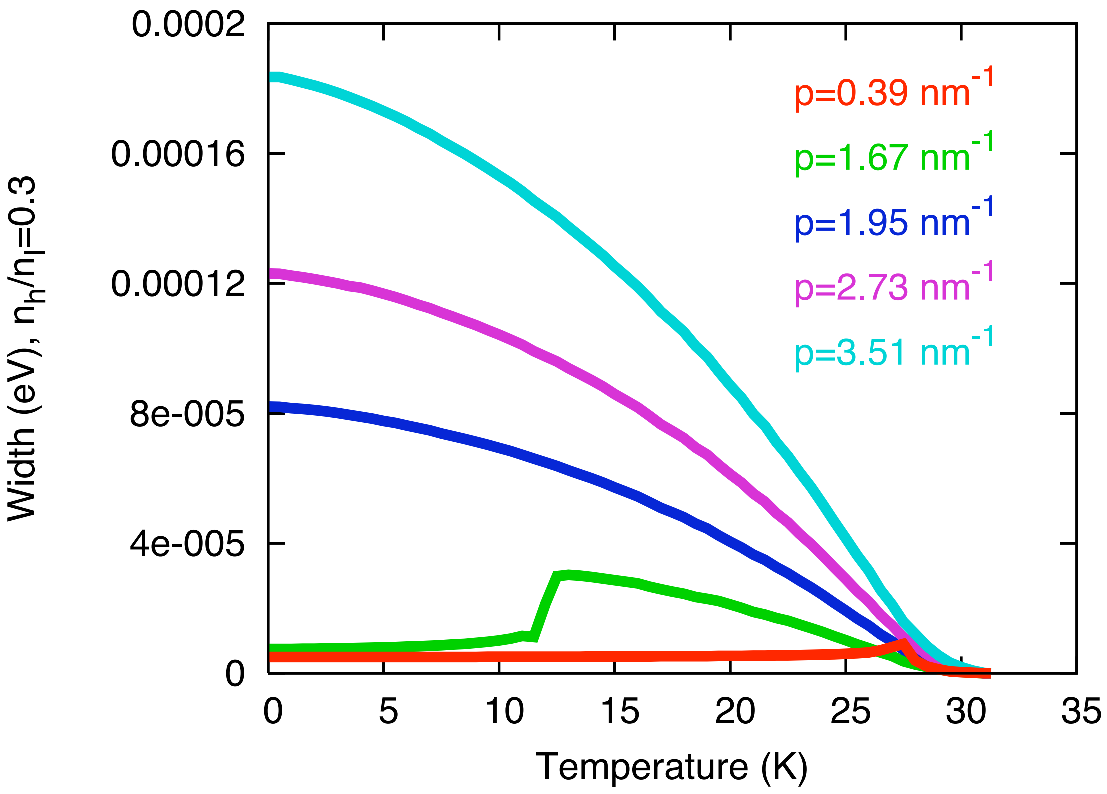


It is rather surprising that the spin-wave relaxation rate shows a significant peak around the Curie temperature!!

Anomalous T dependence (II)



For the density ratio $n_h/n_l=0.3$, the magnetization curve becomes normal.



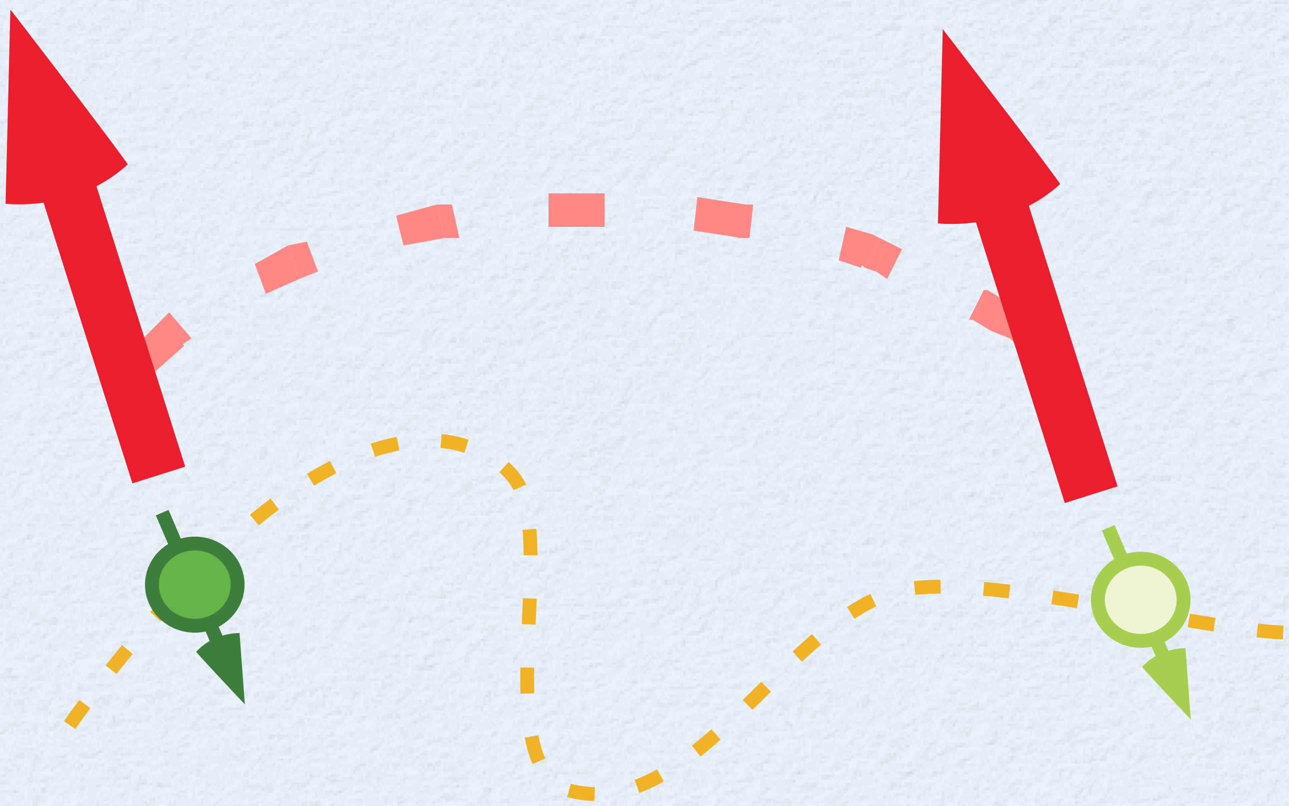
The peak near the Curie temperature disappears as well!!

NON-COLLINEAR EXCHANGE COUPLING

HOW TWO SPINS TALK...

Carrier-mediated Exchange Coupling:

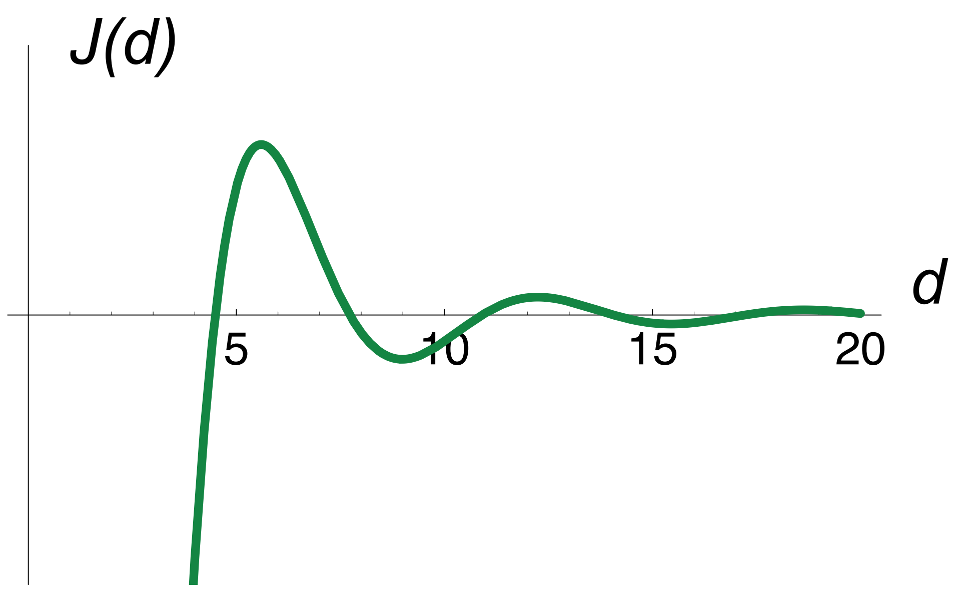
Integrating out the itinerant carriers to derive the effective Hamiltonian for the two spins

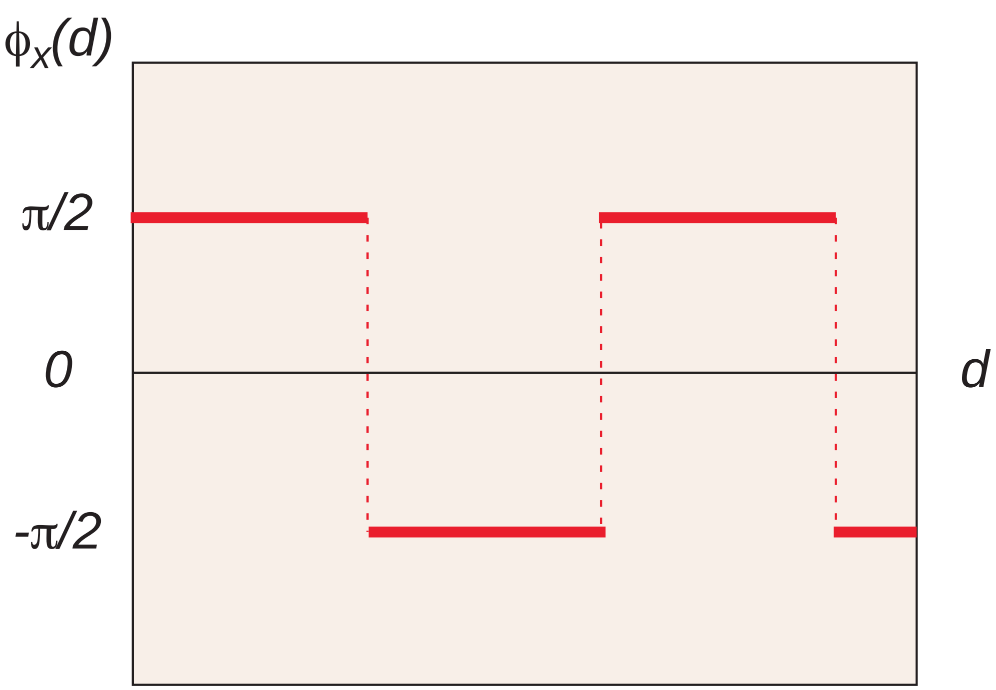


$$H_{\text{eff}} = J(r_{12})\, \boldsymbol{S}(r_1) \cdot \boldsymbol{S}(r_2)$$

Collinear RKKY Instead?

Thus, our conventional wisdom tells us that the mediated effective coupling should have RKKY oscillations…





RKKY interaction:

It can be viewed as the quantum interferences due to patches of the Fermi surface related by the time-reversal symmetry.

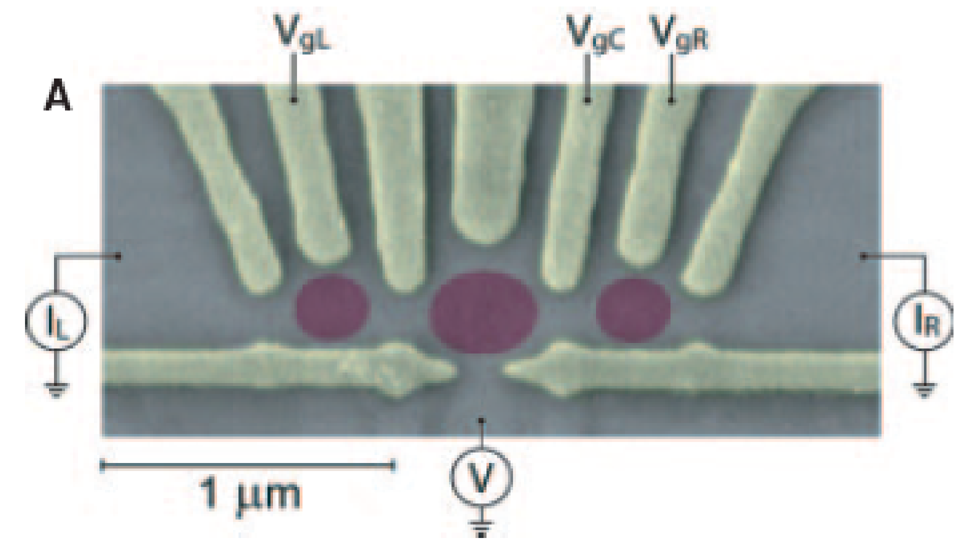
Kondo v.s. RKKY

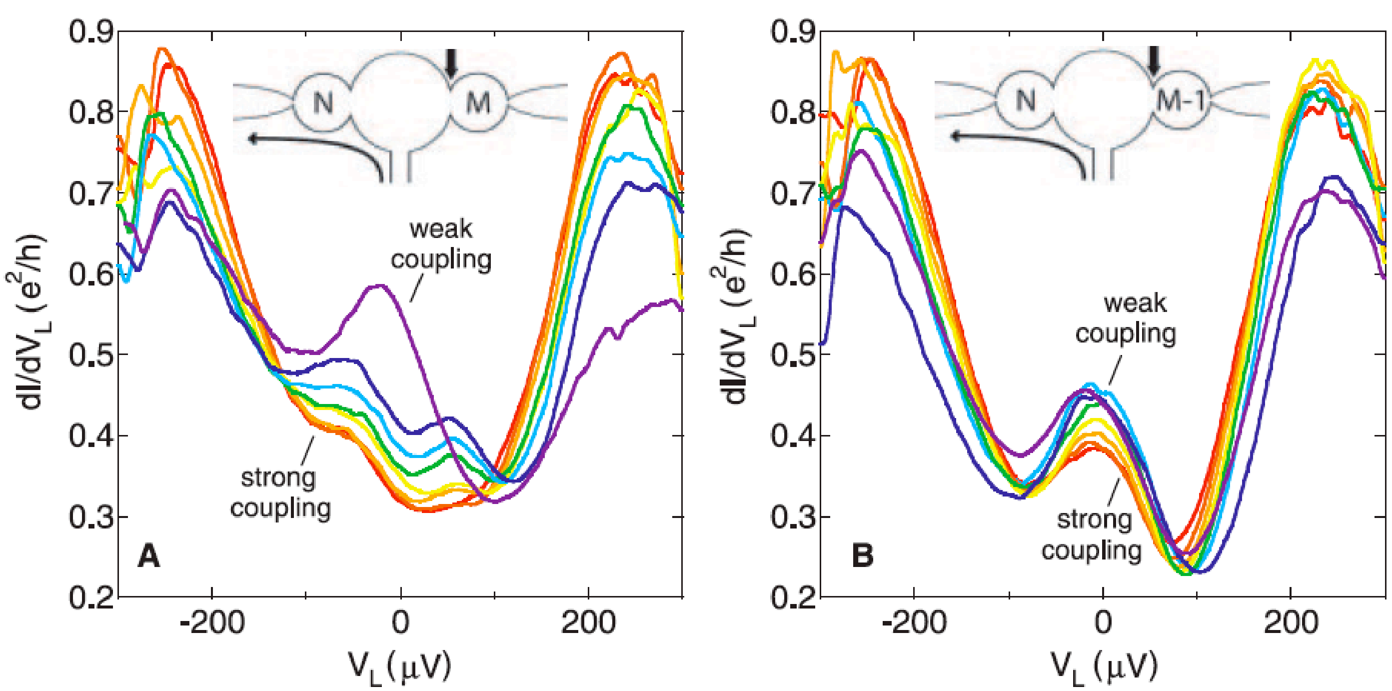
C. M. Marcus' group
Science 304, 565 (2004)

REPORTS

Tunable Nonlocal Spin Control in a Coupled–Quantum Dot System

N. J. Craig,[1] J. M. Taylor,[1] E. A. Lester,[1] C. M. Marcus,[1*]
M. P. Hanson,[2] A. C. Gossard[2]





(a) carrier-mediated RKKY exchange coupling,

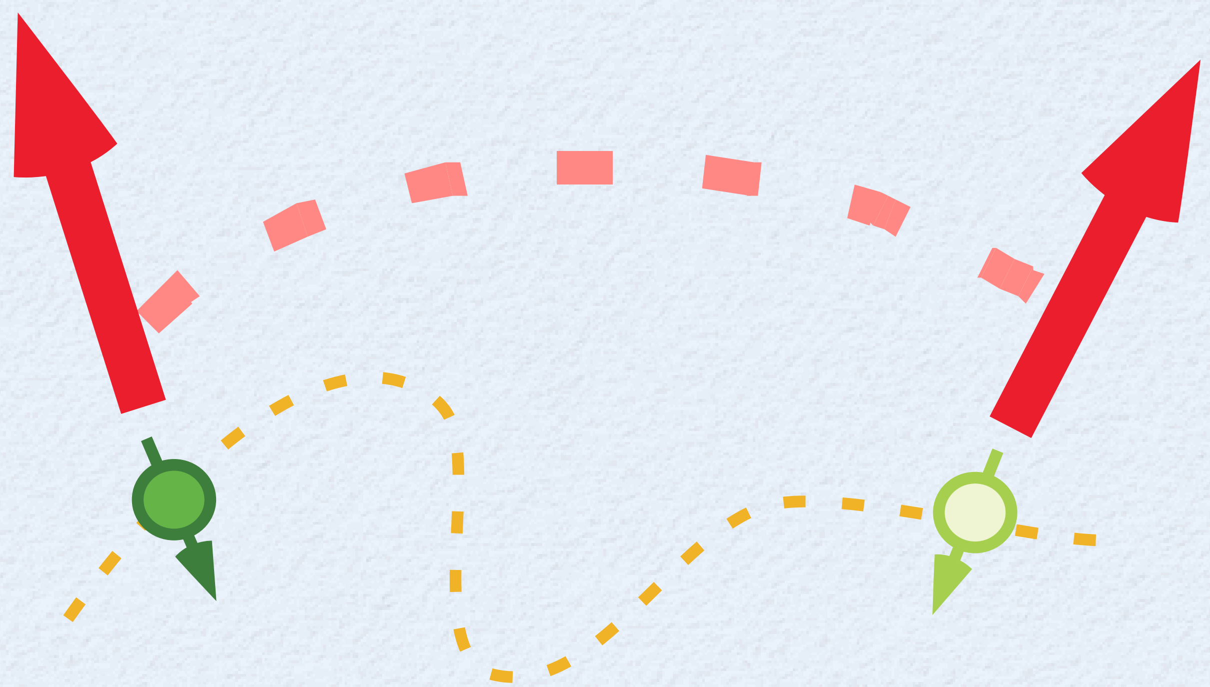
(b) competition between Kondo screening and RKKY interaction,

(c) evolution of the tunneling conductance peak.

NONCOLLINEAR ONE?

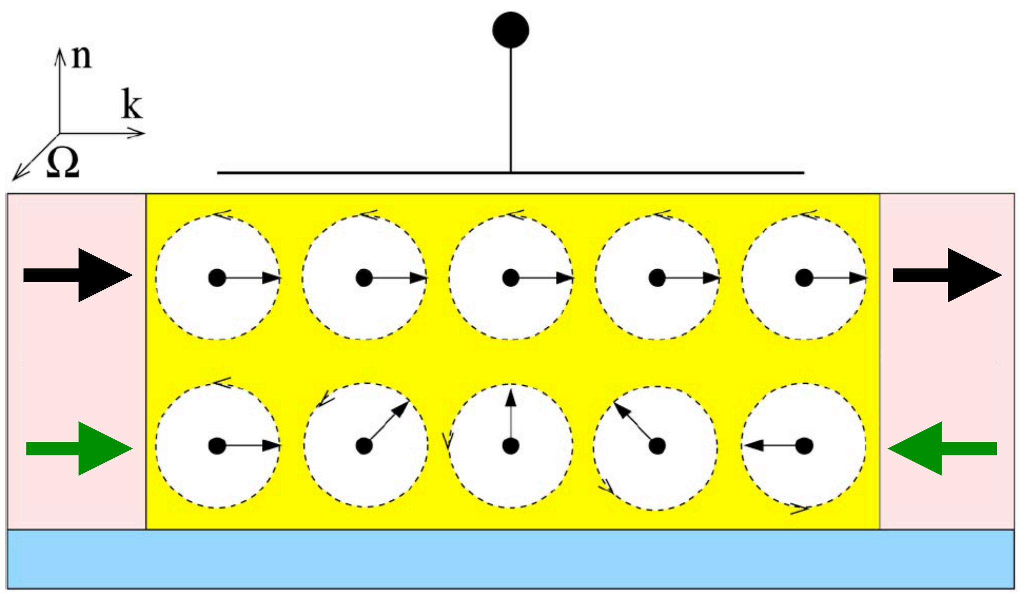
Carrier-mediated Exchange Coupling:

Integrating out the itinerant carriers to derive the effective Hamiltonian for the two spins

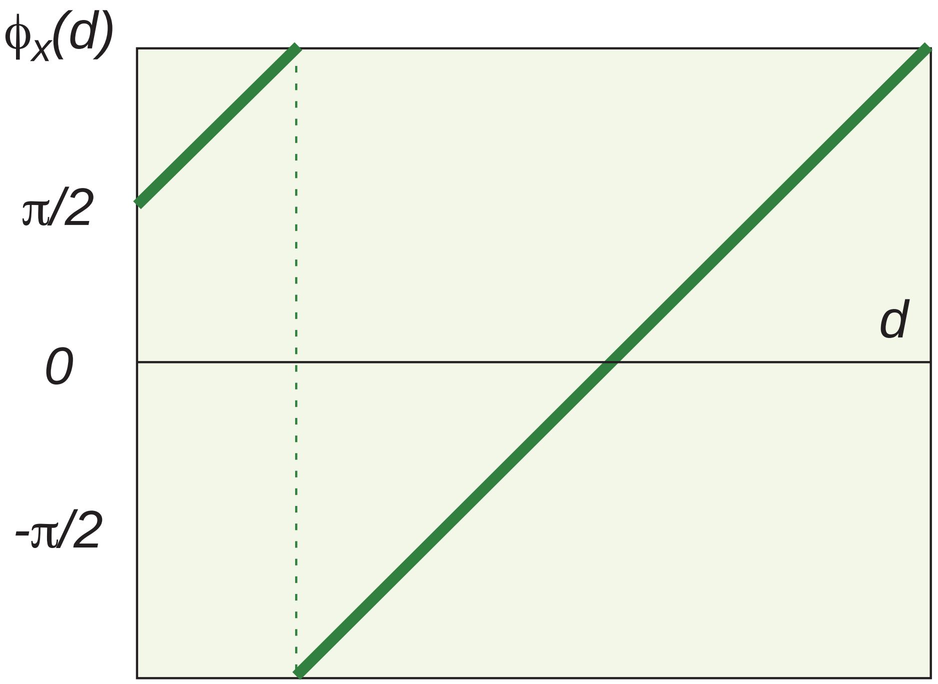


$$H_{\text{eff}} = \sum_{a,b} J_{ab}(r_{12})\, S_a(r_1)\, S_b(r_2)$$

Non-Collinear Spiral Angle?



The spin of the itinerant carriers will align the soft magnet with the same spiral angle.



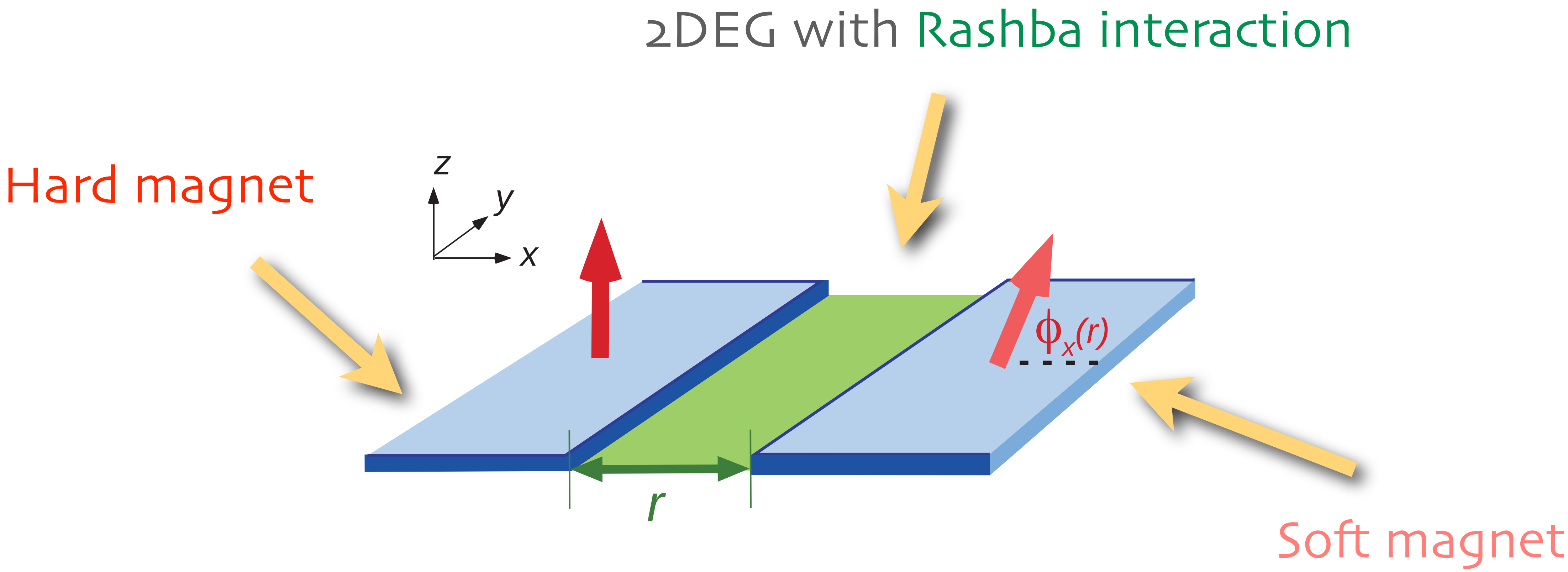
As the spins of the itinerant carriers precess, it is possible to mediate noncollinear exchange coupling.

We try out the idea for Zeeman Hamiltonian and it seems to work…

Therefore, we expect an effective ***non-collinear*** exchange coupling!

Appl. Phys. Lett. 84, 2862 (2004)
Appl. Phys. Lett. 89, 032503 (2006)

Trilayer Magnetic Junction

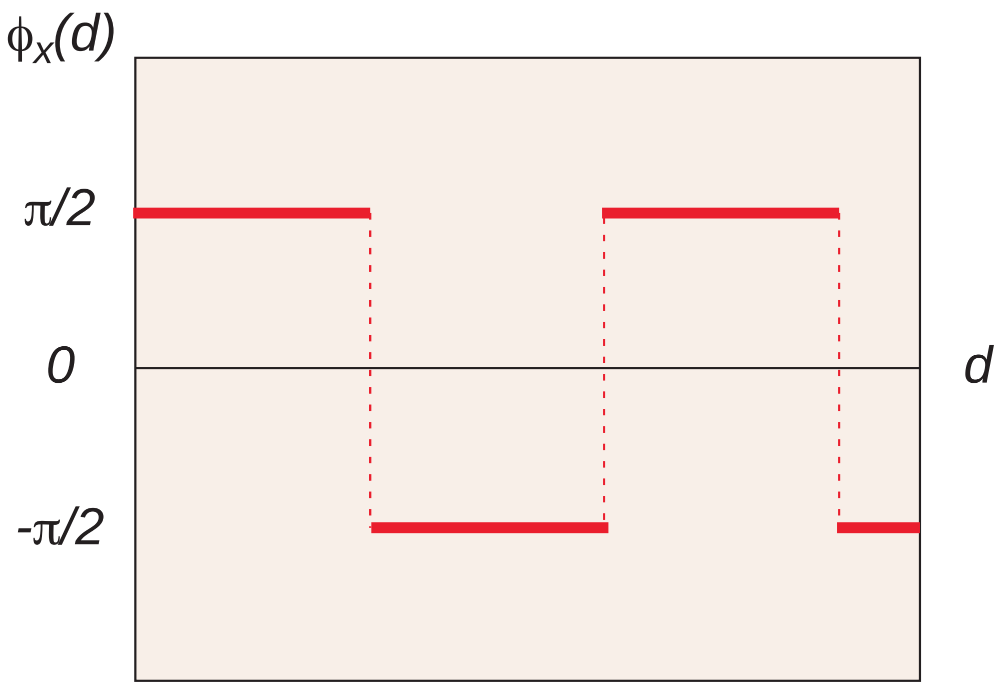


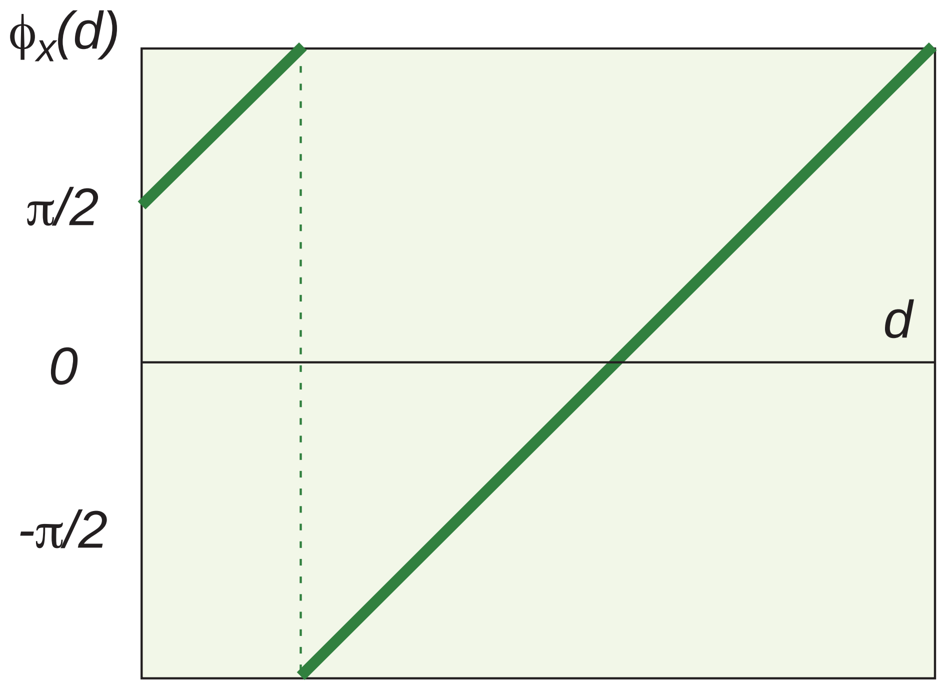
we model the intermediate layer by the Rashba Hamiltonian,

$$H = \int d^2r \, \Psi^\dagger \left[\frac{k^2}{2m^*} \mathbf{1} + \gamma_R (k_y \sigma^x - k_x \sigma^y) \right] \Psi,$$

where γ_R is the strength of the Rashba interaction.

Which one is correct?



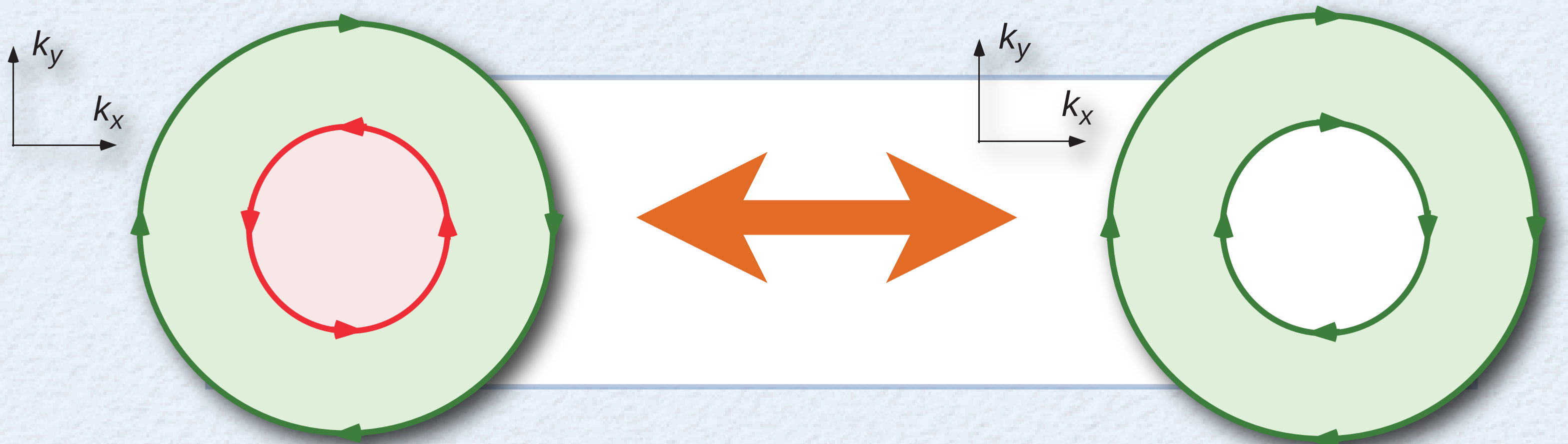


RKKY?

Spiral?

Fermi surface topology

By changing the density, the Fermi surface topology changes as well.

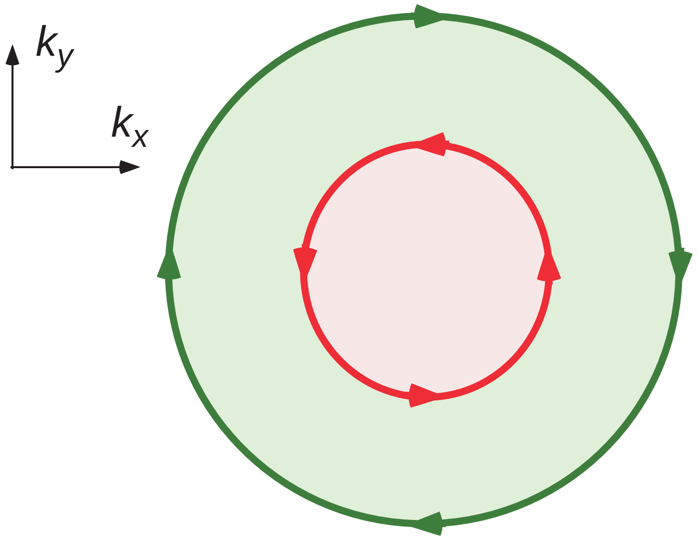


Rashba Hamiltonian for Dummies...

Due to the spin-orbital interaction, spin is no longer the good quantum number but replaced by the chirality instead,

$$\lambda = (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{s}}) \cdot \hat{\boldsymbol{z}} = \pm 1.$$

It is important to remind the readers that, under the time reversal transformation, both momentum and spin reverse their directions and make the chirality invariant.

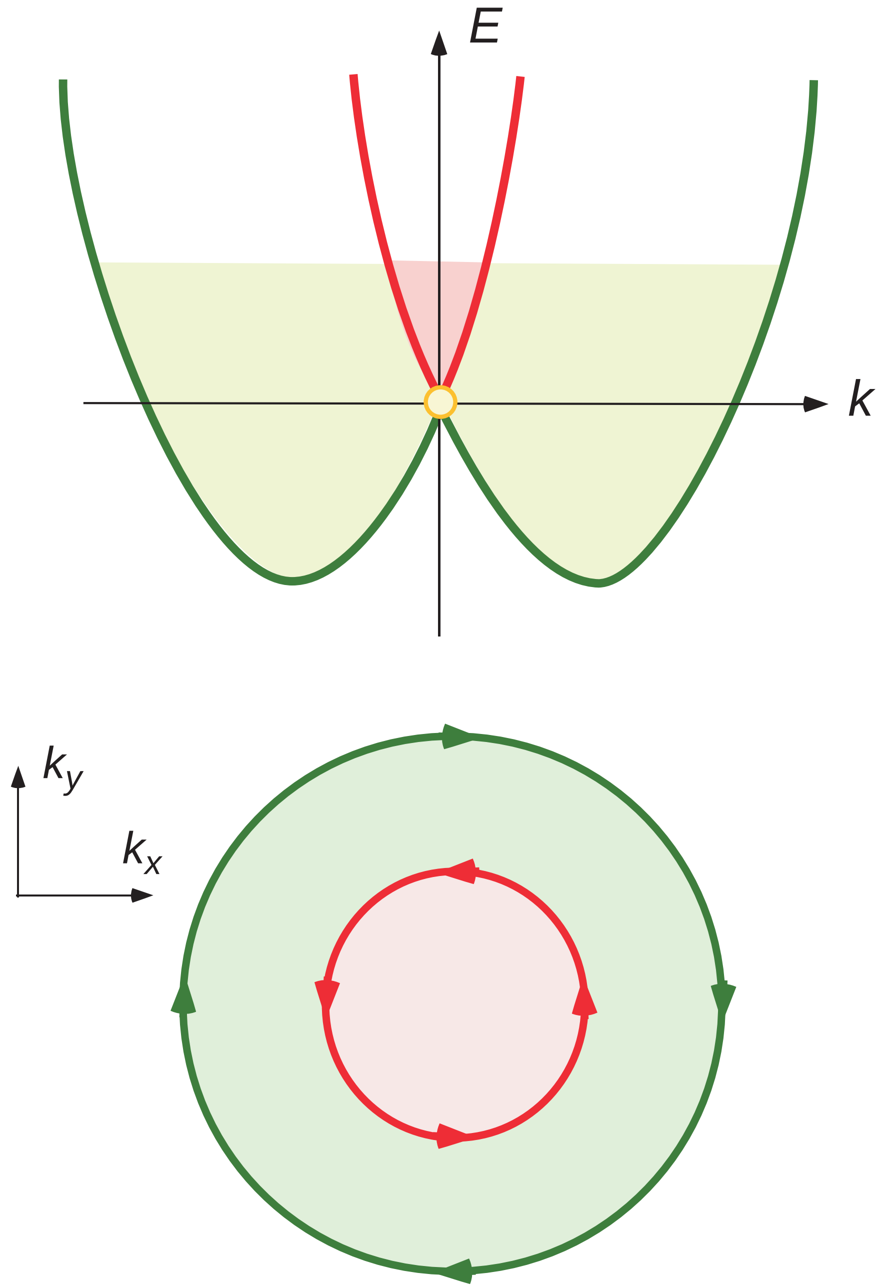


The Rashba Hamiltonian can be brought into its eigenbasis in momentum space,

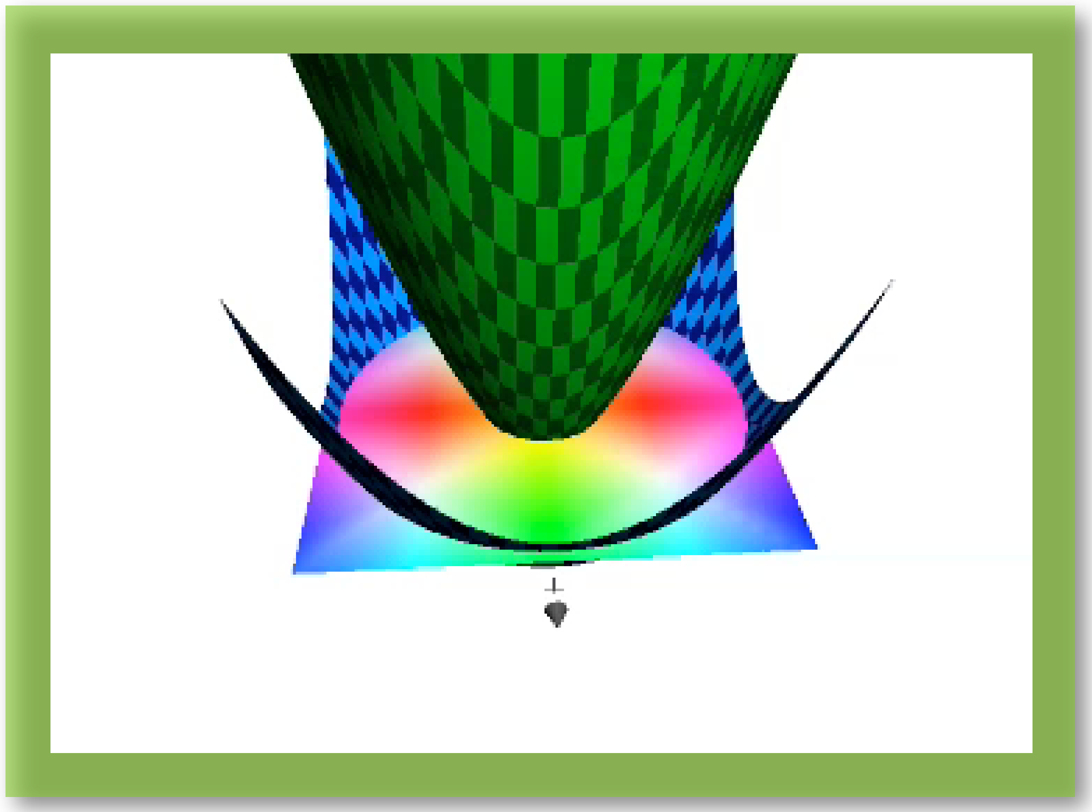
$$\varphi_{k\lambda}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_\lambda(\phi) = \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{2}} \begin{pmatrix} -i\lambda e^{-i\theta_k} \\ 1 \end{pmatrix}$$

where $\theta_k = \tan^{-1}(k_y/k_x)$ with dispersion $\boxed{\epsilon_{k\lambda} = k^2/2m^* - \lambda\gamma_R k.}$

FS Topology 1: Wedding Cake



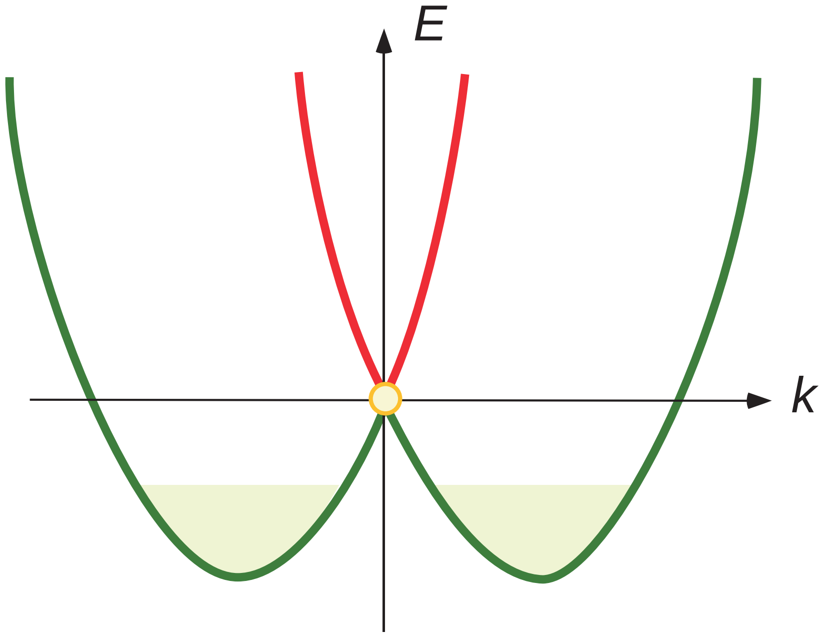
When Rashba coupling is small (compared with the Fermi energy), the Fermi surfaces consist of two particle-like circles with opposite chiralities.

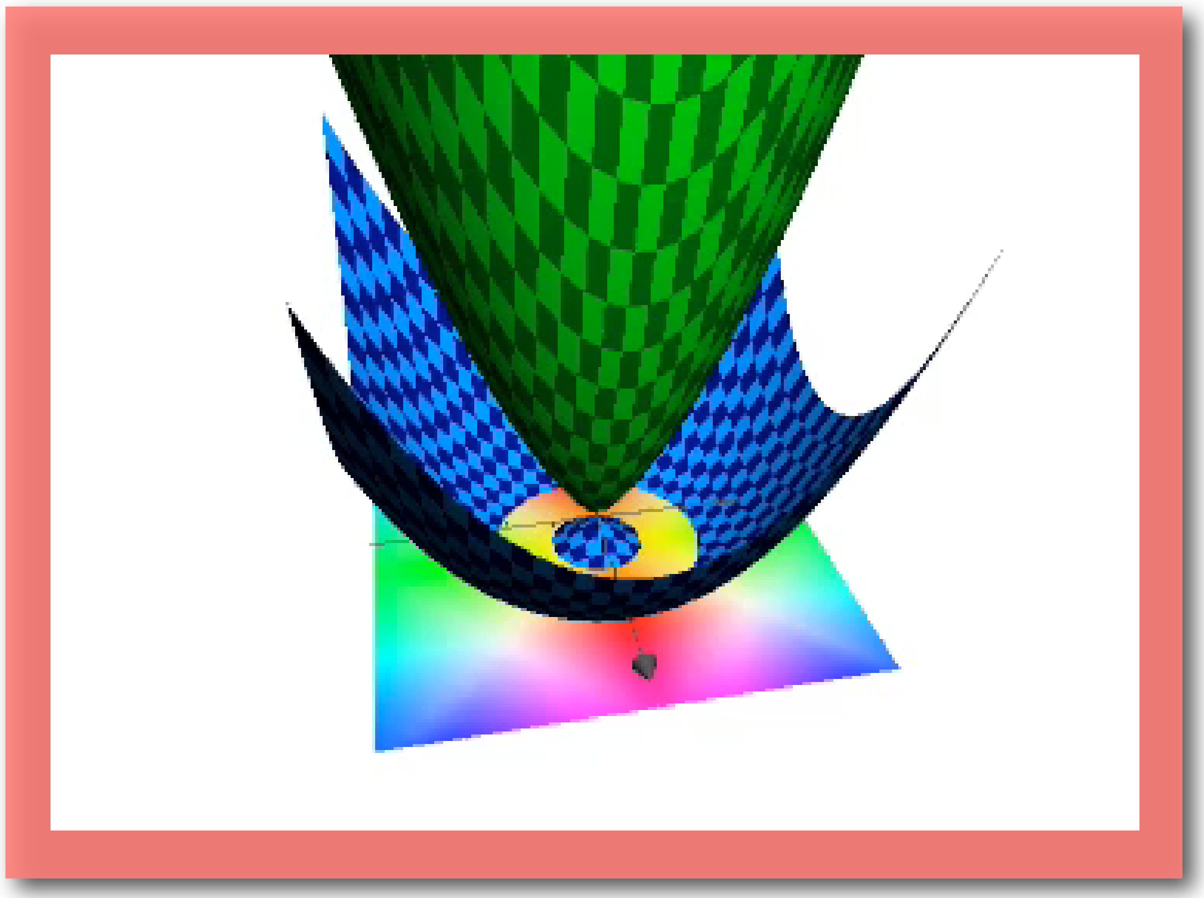


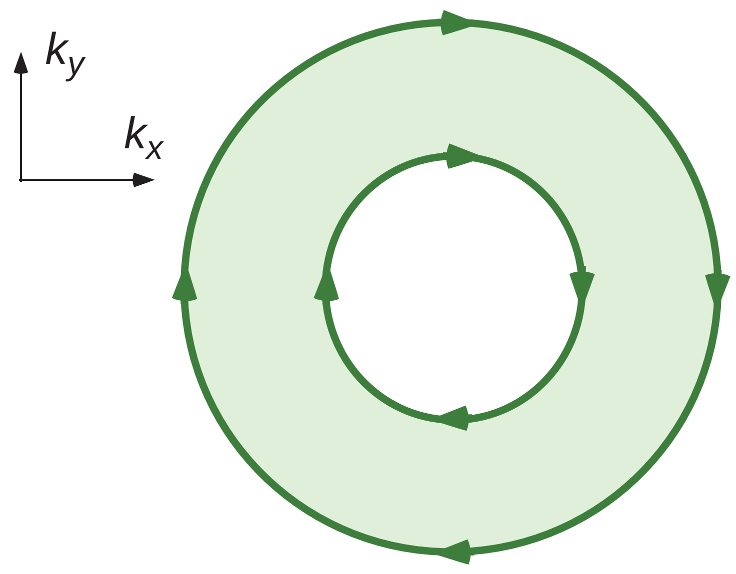
Weak Rashba regime with $\Delta_R/\epsilon_F < 1$

FS Topology 2: Bagel

When the Fermi energy is small, the Fermi surfaces consist of one particle-like and one hole-like circles with the same chiralities.







Dilute density regime with $\Delta_R/\epsilon_F > 1$

Noncollinear Exchange Coupling

Phys. Rev. B 73, 241307(R) (2006)

(1) Integrate out the itinerant carriers → the effective Heisenberg Hamiltonian between the ferromagnets, $H_{\text{eff}} = \sum_{ij} J_{ij} S_L^i S_R^j$.

(2) Within the linear response theory, J_{ij} is proportionally to the spin susceptibility tensor,

$$\chi_{ij}(\vec{r}) = \int_0^\infty dt \left\langle\left\langle i\left[\sigma^i(\vec{r},t), \sigma^j(0,0)\right]\right\rangle\right\rangle e^{-\eta t}.$$

(3) Transforming into the eigenbasis, the susceptibility tensor can be expressed as summations of the product of a weight function and the particle-hole propagator over all possible quantum numbers,

$$\chi_{ij}(\vec{r}) = \sum_{k_1\lambda_1} \sum_{k_2\lambda_2} W_{ij}(\vec{r}) \left[\frac{f(\epsilon_{k_1\lambda_1}) - f(\epsilon_{k_2\lambda_2})}{\epsilon_{k_2\lambda_2} - \epsilon_{k_1\lambda_1} - i\eta}\right].$$

The weight function is $W_{ij}(\vec{r}) = (u^\dagger_{\lambda_1}\sigma^i u_{\lambda_2})(u^\dagger_{\lambda_2}\sigma^j u_{\lambda_1})e^{i(\vec{k_2}-\vec{k_1})\cdot\vec{r}}$, and $\epsilon_{k\lambda} = k^2/2m^* - \lambda k\gamma_R$ is the dispersion for the particle with momentum k and chirality λ.

Symmetries I

Let's take $\chi_{xy}(\vec{r}) = \chi_{xy}(r,\theta)$ as a working example.

(1) **Rotational Symmetry:** Since the operators σ_x, σ_y carry $m = \pm 1 \rightarrow \chi_{xy}(r,\theta)$ contains linear combinations of $m = 0, \pm 2$,

$$\chi_{xy}(r,\theta) = f_0(r) + f_2(r)\cos 2\theta + g_2(r)\sin 2\theta.$$

(2) **Parity Symmetry:** Furthermore, applying the parity symmetry in y direction, it requires $\chi_{xy}(r,\theta) = -\chi_{xy}(r,-\theta)$ and enforces the functions $f_0(r), f_2(r)$ to vanish,

$$\chi_{xy}(r,\theta) = g_2(r)\sin 2\theta.$$

(3) **Time-Reversal Symmetry:** Finally, the Onsager relation from the time-reversal symmetry indicates $\chi_{yx}(\vec{r}) = \chi_{xy}(-\vec{r})$,

$$\chi_{xy}(r,\theta) = \chi_{yx}(r,\theta) = g_2(r)\sin 2\theta.$$

Symmetries II

Utilizing the rotational SO(2), parity P_y (or equivalently P_x), and time reversal symmetries, one can work out the remaining components of the susceptibility tensor,

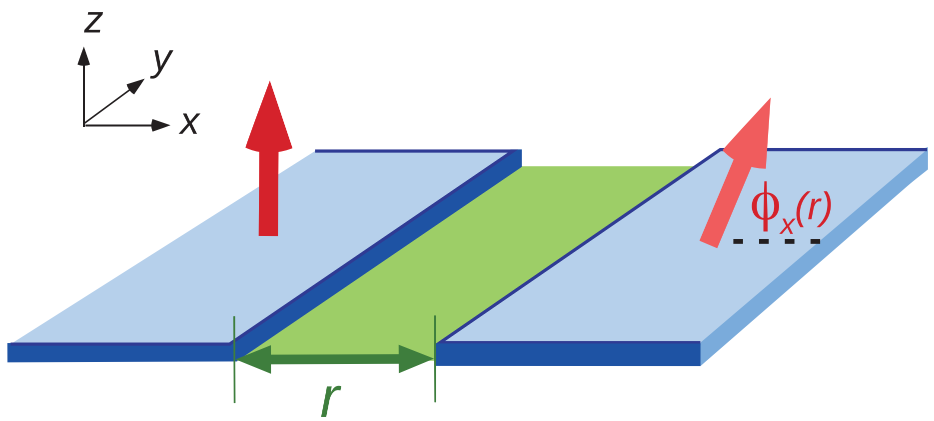
$$\chi_{ij}(r,\theta) = \begin{bmatrix} g_0 + g_2 \cos 2\theta & g_2 \sin 2\theta & g_1 \cos\theta \\ g_2 \sin 2\theta & g_0 - g_2 \cos 2\theta & g_1 \sin\theta \\ -g_1 \cos\theta & -g_1 \sin\theta & h_0 \end{bmatrix}.$$

It is rather remarkable that the symmetry arguments reduce the numerical task down to evaluation of FOUR real scalar functions, $g_0(r)$, $g_1(r)$, $g_2(r)$, $h_0(r)$.

The Rashba Hamiltonian we study here further constrains $h_0(r) = g_0(r) + g_2(r)$, which reduces the number down to THREE.

Spiral Angle

$$\chi_{ij}(r,\theta) = \begin{bmatrix} g_0 + g_2 \cos 2\theta & g_2 \sin 2\theta & g_1 \cos\theta \\ g_2 \sin 2\theta & g_0 - g_2 \cos 2\theta & g_1 \sin\theta \\ -g_1 \cos\theta & -g_1 \sin\theta & g_0 + g_2 \end{bmatrix}$$

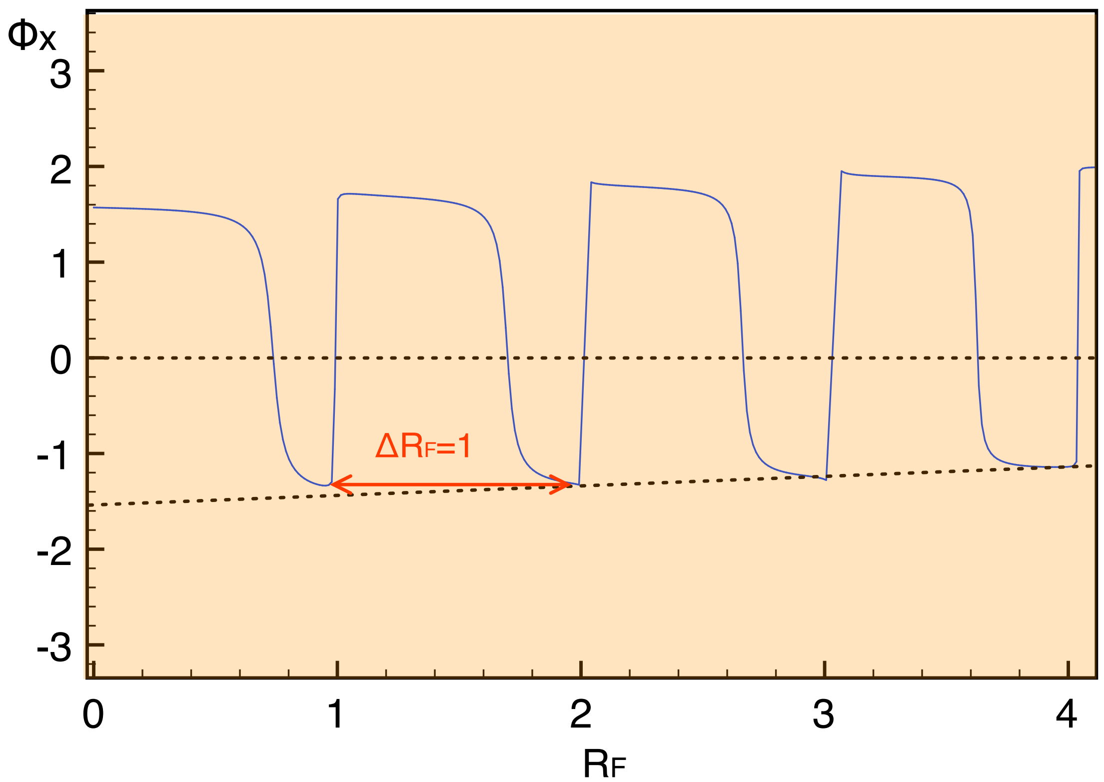


Suppose the ferromagnet on the left of the TMJ is aligned along the z-axis, we are interested in the mediated non-collinear exchange coupling proportional to $\chi_{iz}(r,0)$.

Since $\chi_{yz}(r,0) = 0$, the orientation of the induced moment is captured by the spiral angle,

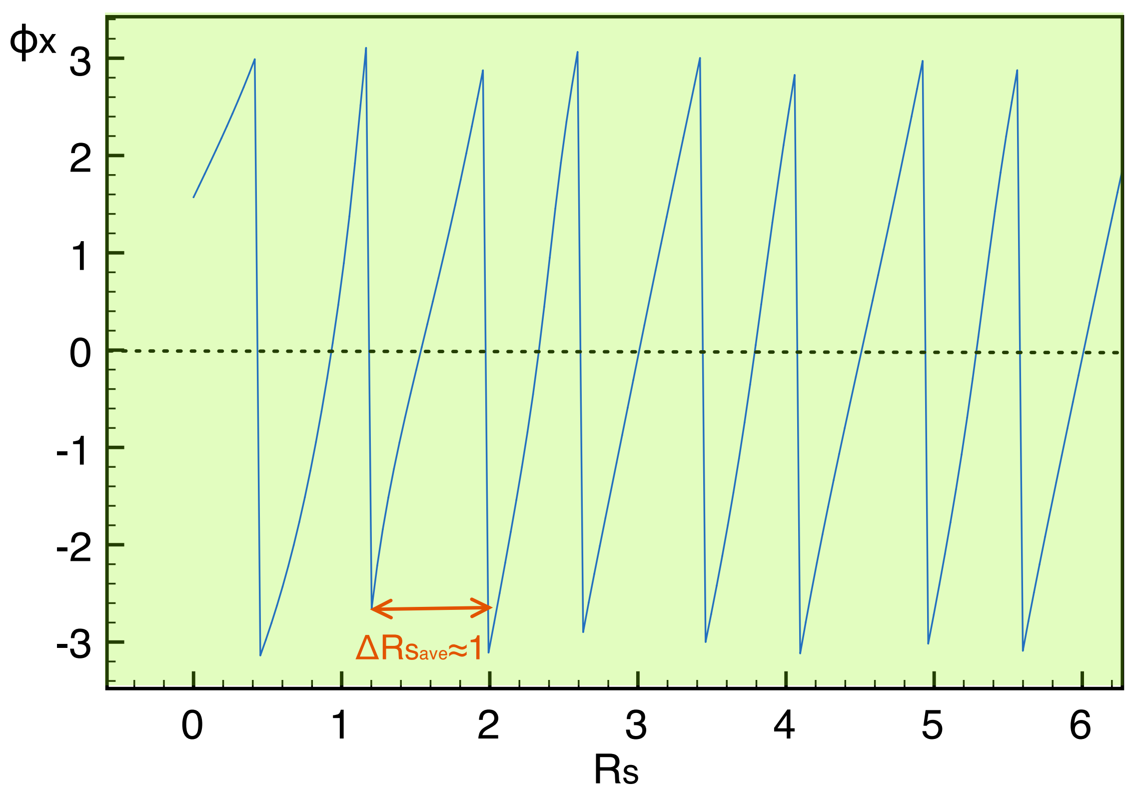
$$\phi_x(r) = \tan^{-1}\left[\frac{\chi_{zz}(r,0)}{\chi_{xz}(r,0)}\right] = \tan^{-1}\left[\frac{g_0(r)+g_2(r)}{g_1(r)}\right].$$

Numerical Results



Weak Rashba regime with $\Delta_R/\epsilon_F < 1$

Robust spiral backbone with minor oscillatory residues resembling the RKKY oscillations.



Dilute density regime with $\Delta_R/\epsilon_F > 1$

Modified RKKY oscillation with a gradual upwinding trend due to Rashba interaction.

Connection to 1D Rashba?

(1) In the asymptotic limit $k_F r \gg 1$, the reduced spin susceptibility along the radial direction $\chi_{ab}(r)$, where $a, b = x, z$, can be well approximated as 1D Rashba system.

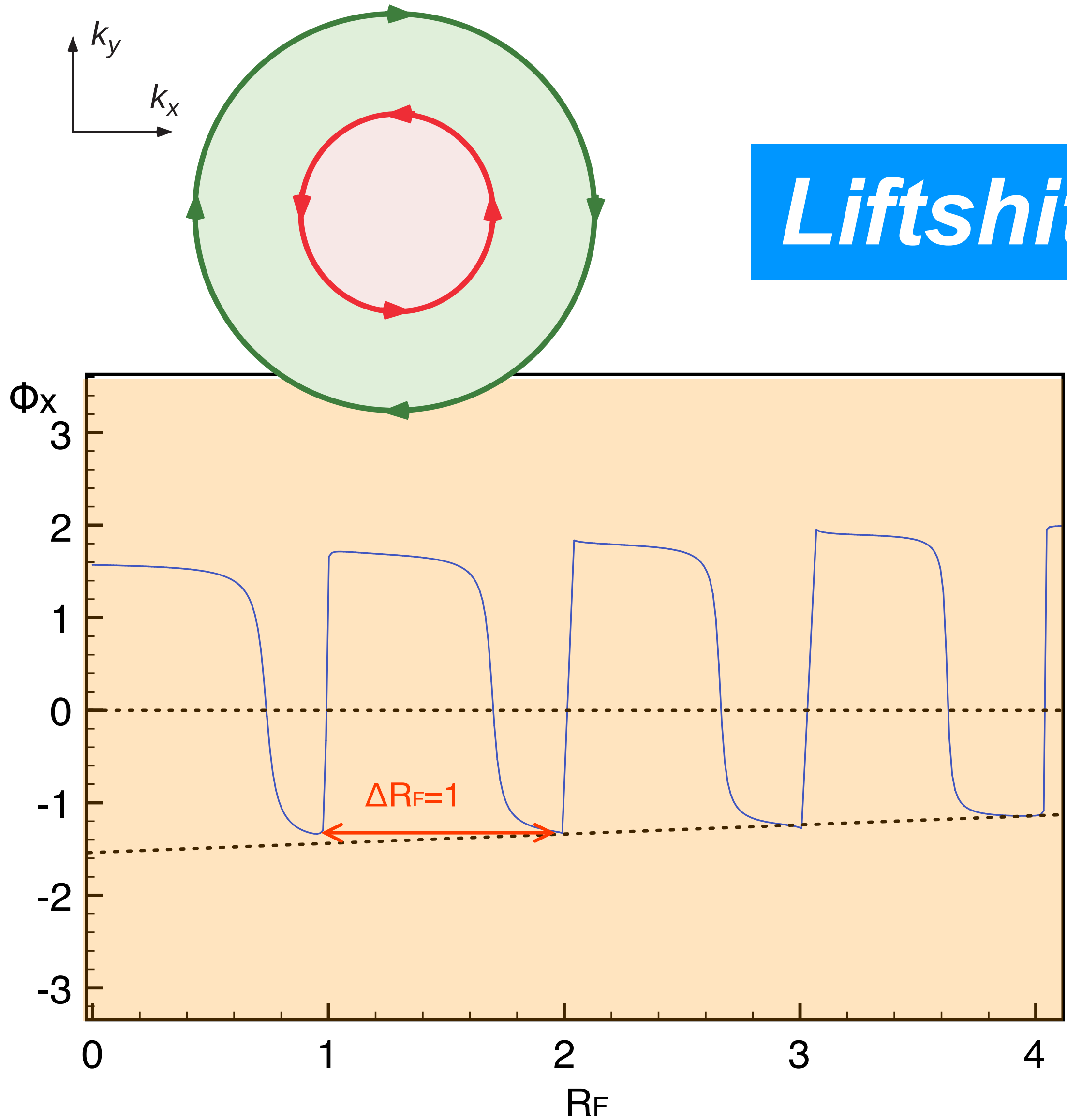
(2) Applying a local gauge transformation, $U(r) = e^{-ik_R r \sigma^y/2}$, the Rashba Hamiltonian can be mapped into the 1D free electron gas with the well-known RKKY spin susceptibility.

(3) Since the local gauge transformation is nothing but the local rotation about the y-axis with the spiral angle $\phi(r) = k_R r$, the reduced susceptibility is approximately the usual RKKY oscillation twisted by a local spiral transformation,

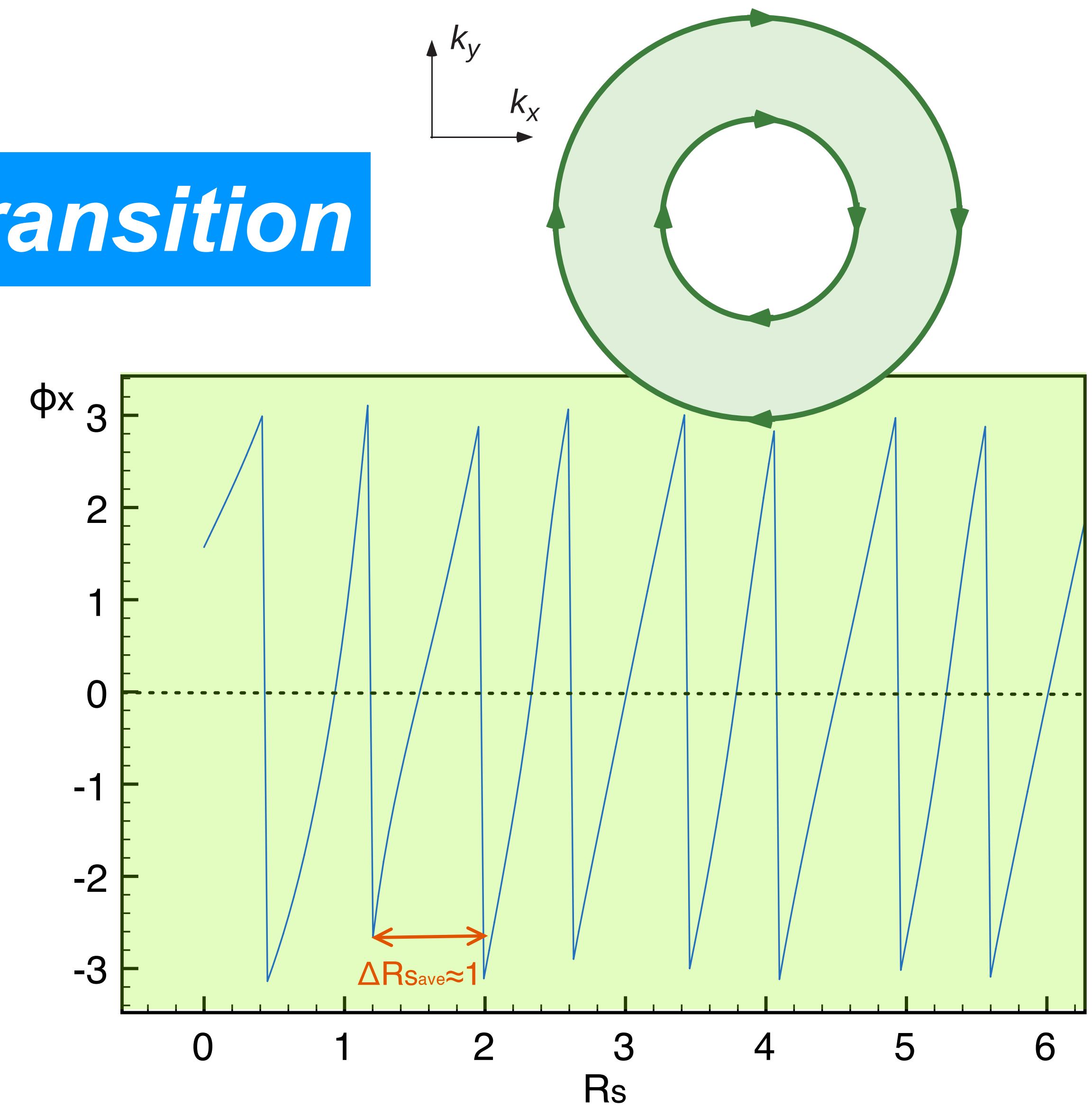
$$\chi_{ab}(r) \approx \sum_c \begin{bmatrix} \cos k_R r & -\sin k_R r \\ \sin k_R r & \cos k_R r \end{bmatrix}_{ac} \chi_{cb}^{RKKY}(r).$$

Fermi Surface Topology

By changing the carrier density, we can change the topology of the Fermi surfaces from wedding cake (with opposite chiralities) to bagel (with one chirality).

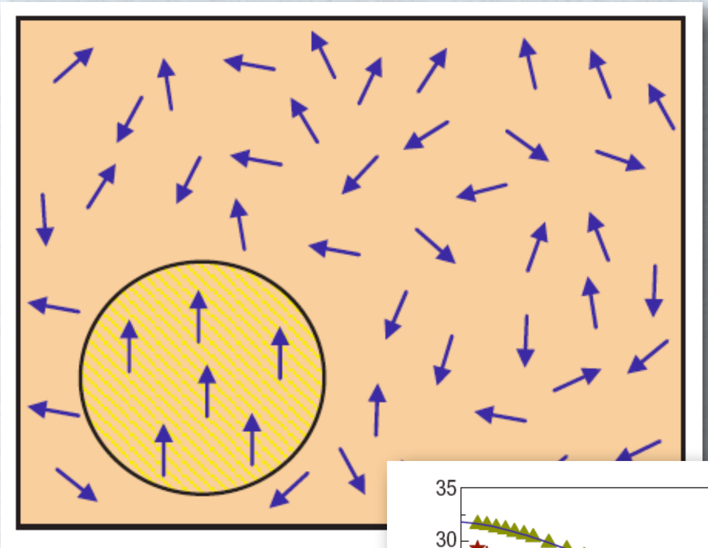


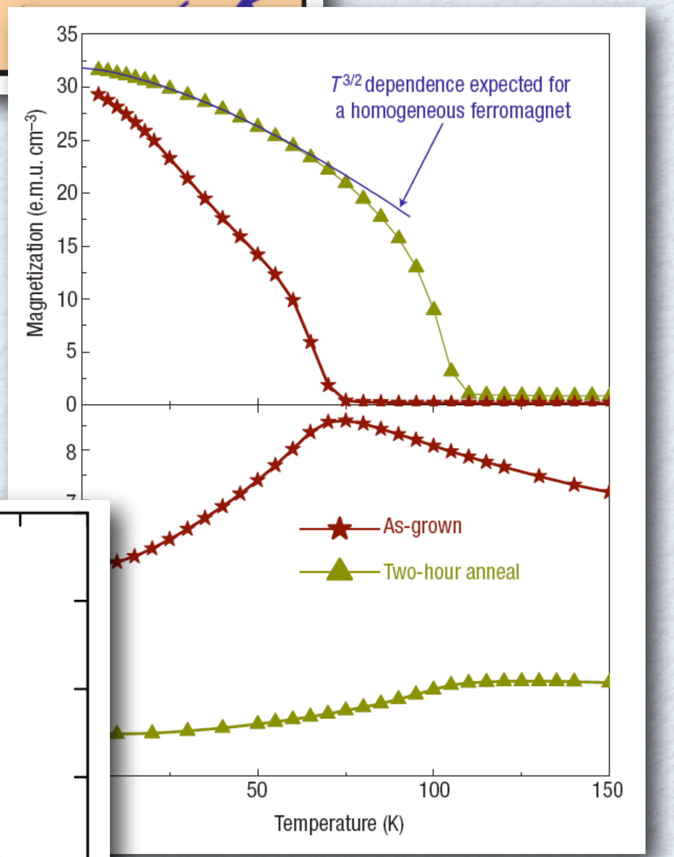
Liftshitz Transition

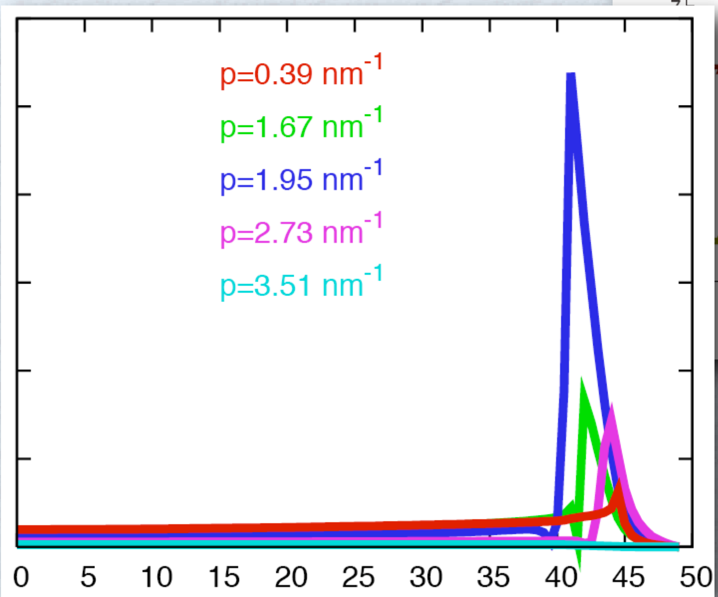


SUMMARY I

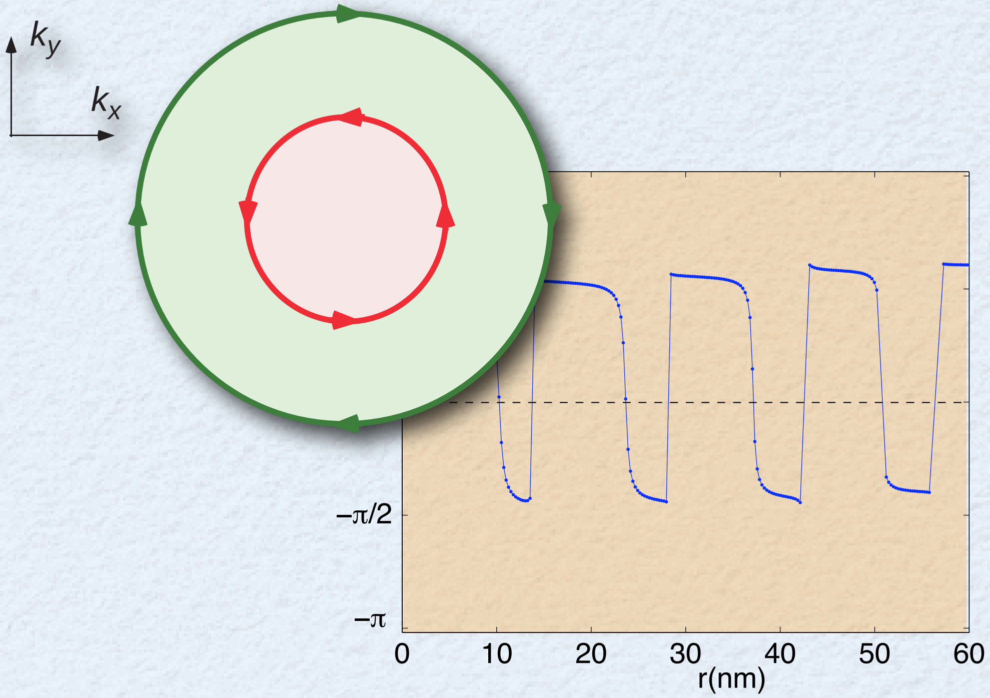
- Carrier-mediated ferromagnetism in diluted magnetic semiconductor.
- Self-consistent Green's function approach for spin-wave dynamics.
- Strong correlation between magnetization curve and spin relaxation rate.
- More needs to be done ...



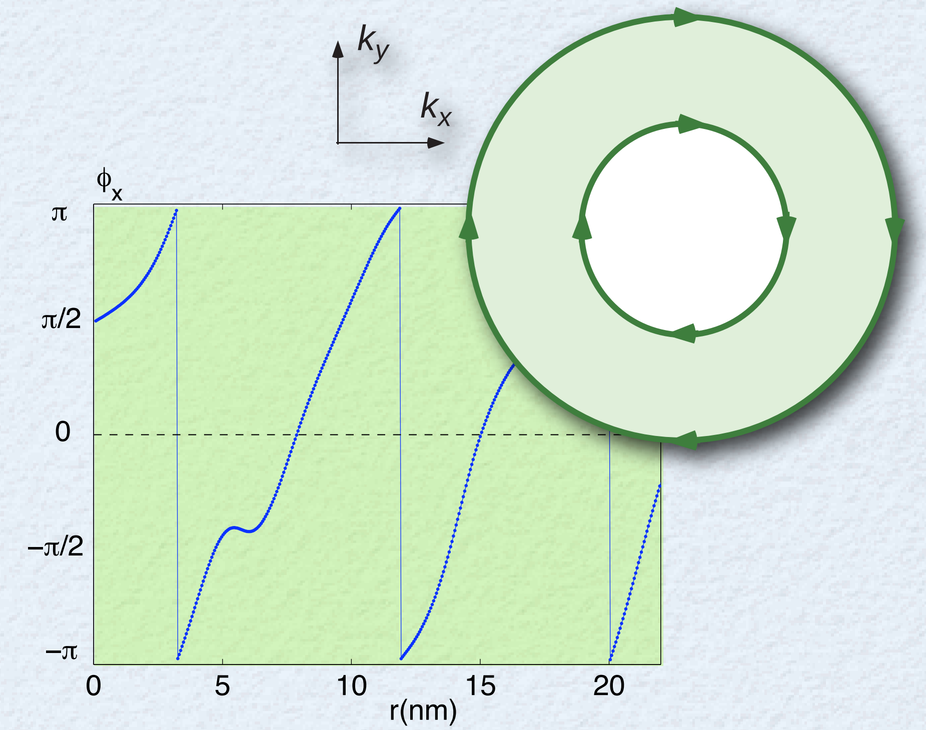




SUMMARY II



- Non-collinear exchange coupling mediated by Rashba interaction.
- RKKY or Spiral? Depending on the Fermi surface topology.
- Liftshitz transition by changing the carrier density in the 2DEG.
- Potential applications? Ballistic vs diffusive? & Lots of open questions...



THANK YOU!!